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NUMERICAL MODEL FOR DETERMINING THE WETTING FRONT IN A CLAY LAYER OF A LEAKING COMPOSITE BARRIER

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ABSTRACT

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Composite barriers for hazardous and solid waste landfills are designed to minimize hydraulic transport of the contaminant leachate of the landfill; however, the geomembrane can be damaged during construction or otherwise contain imperfections from manufacturing. It is desirable to estimate the discharge characteristics and the flow width of such a leachate should leaks in the geomembrane occur.

Numerical solutions of the flow equations for unsaturated soil barriers can provide an understanding of the movement of a leachate through a composite barrier. The purpose of this investigation was to develop a computer model that simulated two-dimensional migration of leachate beneath holes in a geomembrane in the composite barrier profile. The solution of this problem aids the determination of the time required for a wetting front to reach the leachate detection system. An approximate model of a composite barrier with a damaged geomembrane was considered. The top of the geomembrane and bottom of the leachate detection system were the boundaries for simulation, and then the soil profile was analyzed. Finally, finite elements were utilized accounting for leachate placed on top of the clay barrier. The flow through the soil below a slit was computed using Richards' equation.

A computer code to evaluate leakage through composite barriers was presented. The code, COMPBAR (composite barrier), simulates highly nonlinear, two-dimensional movement of leachate in unsaturated composite barriers with damaged geomembranes. A finite element approximation of Richard's equation describing flow through unsaturated porous media was incorporated. Galerkin's method and Picard iteration were utilized to solve Richard's equation. The governing equation reflected barrier heterogeneity and the dependence of hydraulic properties on the degree of saturation. The time required for the wetting front to reach the leachate detection layer and the time necessary for a stationary flow to develop in the soil barrier were computed.

Outputs of the developed COMPBAR program compared well with outputs of the SOILINER and SEEP programs. Design charts and guidelines were developed that can be used by designers to estimate the effective life of composite barriers and to aid study of various parameters that affect wetting front movement in the composite barrier.

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CHAPTER 1
INTRODUCTION

The possibility of groundwater contamination by a landfill depends on many parameters. The two most significant factors are the amount of leachate migration out of the landfill and the corresponding contaminant concentrations. The first component is controlled by design, construction, and maintenance of the landfill, while the second component depends on the waste characteristics, available moisture, and mean annual temperature of the landfill site. Transient rates of leakage through the soil profile depend on previous moisture conditions, soil properties, the soil barrier, and the rainfall intensity.

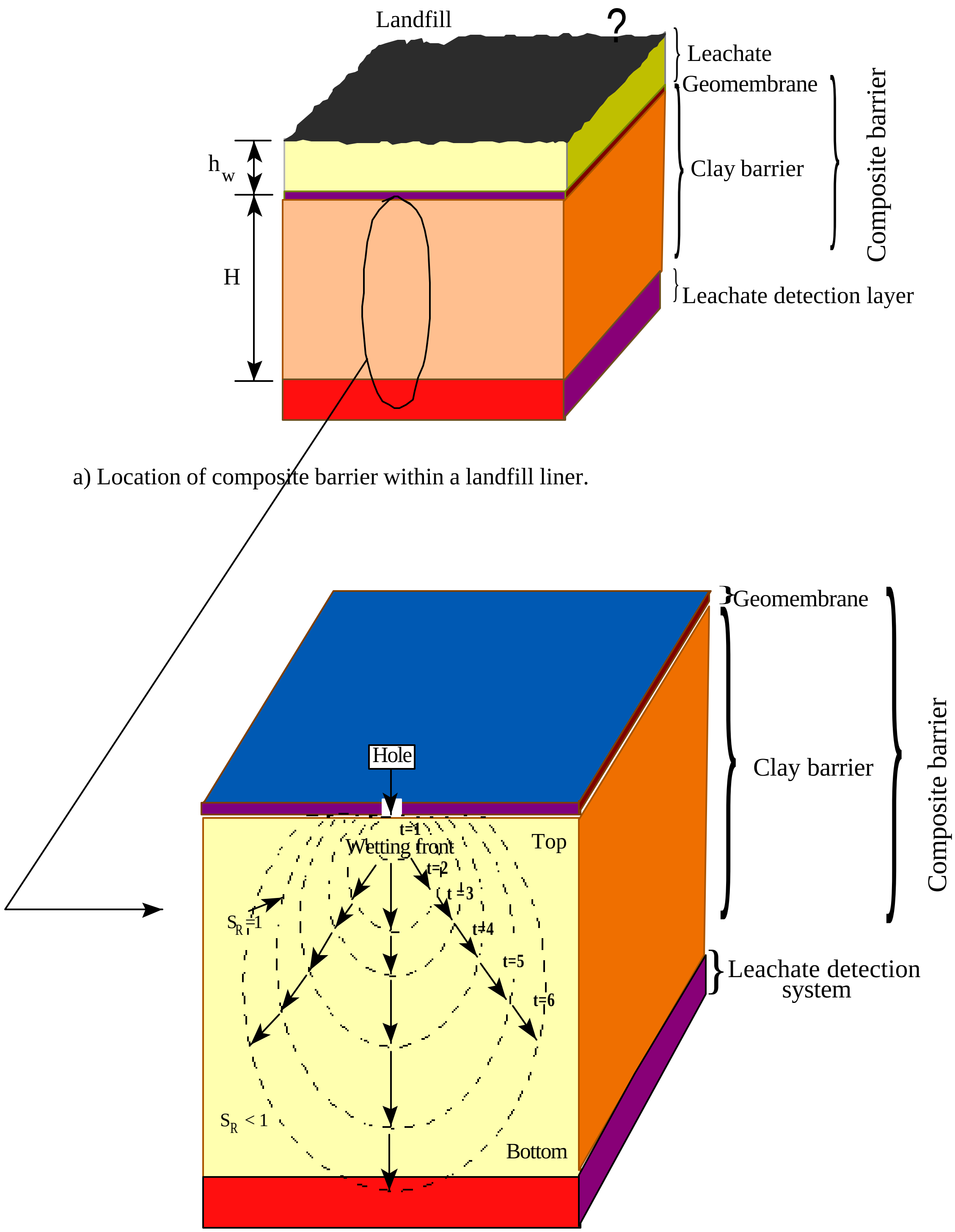
New hazardous and municipal solid waste landfill liners are built with a composite barrier. A composite barrier has two elements, a geomembrane (flexible membrane liner, FML) as the top element and a low hydraulic conductivity material (such as clay or clayey soil) as the bottom element. A sand layer is placed above the FML to facilitate lateral flow of the leachate towards collection pipes. Geomembranes are becoming the most commonly utilized material for lining containment facilities such as landfills, water basins, and chemical or waste lagoons. The use of geomembranes to line waste containment facilities, e.g. hazardous waste landfills and leachate impoundments, is mandatory in some nations (Giroud and Bonaparte 1989b). A geomembrane as a part of the composite barrier is required for all hazardous waste facilities and acts as the primary liner (Miller and Mishra, 1989).

Composite barriers are increasingly used for leachate accumulation and waste control facilities. First, the landfill liner is constructed including the composite barrier (if required). Then the waste is placed in an attempt to reduce leachate generation and potential groundwater contamination. Leakage through a composite barrier can result from flow through geomembrane slits. A geomembrane can be damaged during construction and installation. The objective of the composite barrier system is to use two different barrier materials, where any holes in the geomembrane will be cut off by the underlying clay, making it more difficult for leachate to leak through the holes. Precipitation that infiltrates a solid or hazardous waste landfill can potentially increase the rate and total volume of leachate generated if the water is not collected or drained properly.

There are two primary mechanisms of leakage through geomembranes: fluid permeation through an intact geomembrane or flow through geomembrane punctures, holes or tears (Giroud and Bonaparte 1989b). Leakage rates due to flow through geomembrane holes are usually much higher than the leakage rates due to fluid permeation through intact geomembranes (Giroud and Bonaparte 1989 a, b). If a geomembrane with a hole is placed on a layer of low hydraulic conductivity soil to form a composite barrier, the soil significantly blocks the flow of leachate through the slit, provided that the geomembrane is in close contact with the soil. The mechanism of leakage through a damaged geomembrane with a slit is as follows: the leachate first migrates through the hole in the geomembrane. After that, the leachate may travel laterally some distance in the space between the geomembrane and the soil. Then, the leachate migrates vertically into and, ultimately, through the soil. Finally, the leakage is detected in the leachate detection and collection layer. The schematic representation of leakage through a composite barrier is shown in Figure 1.1.

If a geomembrane with a hole is overlain and underlain by high hydraulic conductivity materials (such as geonet or coarse gravel), flow of the leachate can be considered as free flow through an orifice and the leakage rate is governed by the size of the hole. If a geomembrane contains a hole and is placed on a high hydraulic conductivity material, but is overlain by a sandy or a fine gravel layer, flow through the hole may be somewhat impeded (Giroud and Bonaparte 1989b).

The rate of leakage in the case of a composite barrier is affected by the types of damage in the geomembrane, e. g. , slit hole, pin hole, and punch hole (Brown et al. 1987). Previous research work indicates that a key factor influencing the rate of leakage through a composite barrier is the quality of contact between the geomembrane and low hydraulic conductivity soil (Giroud and Bonaparte 1989 a, b; Giroud et al. 1989). Giroud and Bonaparte (1986) reported experimentaland analytical estimates of the flow through damaged geomembranes. They



a) Location of composite barrier within a landfill liner.

b) Progression of wetting front extending from a hole in the geomembrane through the compacted soil barrier (t=time, S_R = saturation).

Figure 1.1 Schematic diagram of a composite barrier provided a relationship between the size of the leakage beneath the geomembrane and the head of the leachate level above the geomembrane. They also proposed a method of interpolation between the various experimental and theoretical results, which leads to the evaluation of the leak rates through composite barriers.

In the literature, there are several computer programs related to unsaturated or saturated soil profiles, but to date only a few have studied composite barriers with a damaged geomembrane underlain by an unsaturated soil profile. Flow through composite barriers with damaged geomembranes has not been well established. However, Giroud et al. (1989) gave useful guidelines about flow through holes in a geomembrane. The significance of the present investigation is the development of a finite element model to analyze the nonlinear flow of leachate and the progress of a wetting front in a composite barrier with a damaged geomembrane.

Objectives of this research were: (1) to study the progress of a wetting front in a soil barrier without a geomembrane (one-dimensional nonstationary unsaturated flow), (2) to analyze the progress of a wetting front in a composite barrier with a geomembrane damaged during installation at the site or during manufacturing process (two-dimensional nonstationary, unsaturated flow), (3) to predict in both cases, the time in which the wetting front reaches the leachate collection and detection, (4) to develop a finite element computer program that is capable of simulating flow through damaged geomembranes, and (5) to study different composite barrier scenarios with damaged geomembranes which illustrates the usefulness of the developed program.

The finite element program developed to achieve these objectives is called COMPBAR (composite barrier). COMPBAR simulates nonlinear two-dimensional movement of leachate water in unsaturated composite barriers with damaged geomembranes, and one and/or two-dimensional transient nonlinear flow through a soil barrier system. This program can determine the time required for the wetting front to reach the leachate collection layer and estimate the time necessary for stationary flow to develop in the soil barrier. This user-friendly computer program, COMPBAR, was developed to achieve this goal.

The scope of this investigation was limited to computer simulation of the design model. In Chapter 2, Literature Review, soil barrier and composite barrier systems are presented. A review of the literature on numerical modeling of unsaturated soil and the finite element method is also presented. Chapter 3 presents the research plan of this study. In Chapter 4, several example results, which are obtained from the COMPBAR program, and comparisons with SOILINER and SEEP programs are presented. Actually, both of these programs deal with leachate migration through an unsaturated soil. Stability analysis of COMPBAR code and results of a parametric analysis are also provided in this chapter. Conclusions and recommendations are presented in Chapter 5 and Chapter 6, respectively. The COMPBAR computer program (main program and subroutines) with data input and output for Example 1 are presented in Appendix A.

CHAPTER 2
LITERATURE REVIEW

2.1 Overview

A predominant issue of landfills is the potential contamination of groundwater and surface waters. Many controls must be designed and implemented to minimize infiltration through the landfill profile, maximize the leachate detection efficiency of the liner system, and prevent the migration of contaminants into groundwater. Consequently, to evaluate the design of hydraulic barriers, it is important to have design tools that can determine water-balance components and estimate leachate movement into, through, and out of landfill systems. Rainfall that infiltrates solid or hazardous waste landfills increases the rate and total volume of leachate produced. A soil hydraulic barrier or a composite barrier is typically placed below (i.e., liner) and above (i.e., cover) a waste landfill to reduce leachate generation and potential groundwater contamination (Peyton, 1988). Gray (1984) determined that no single convenient lining material appears appropriate for the long-term impoundment of waste materials. Composite barriers are mandated by regulatory agencies. Without the geomembrane water may reach the leachate detection system very rapidly. Application of the geomembrane on top of the soil hydraulic barrier decreases the leakage rate. When a slit forms in the geomembrane, the presence of low hydraulic conductivity soil in intimate contact with the geomembrane decreases the rate of leakage through the slit. A more suitable design strategy would be a performance-based design (Shackelford, 1992).

2.2 Landfill Construction and Its Barrier Systems

A hydraulic barrier is a low hydraulic conductivity material utilized to drastically slow down the flow of liquid. Low hydraulic conductivity materials utilized in civil engineering applications to assemble barriers include: clays, silty clays, clayey sand, silty sand, and geosynthetics such as geomembranes. Typical waste containment liner systems are briefly addressed in the following sections.

2.2.1 Geomembrane Barriers

Geomembranes are extraordinarily low hydraulic conductivity ($1x10^{-13}$ m/s to $1x10^{-14}$ m/s) membrane barriers. They are often used with soils to control fluid movements in a structure or system. Geomembranes are used in the construction of water reservoirs, canals, municipal and hazardous solid waste landfills, leachate impoundments, landfill covers, and spill containment systems. Geomembranes include asphalted membranes and polymeric membranes. Some examples of polymeric membranes are high-density polyethylene (HDPE), linear medium density polyethylene (LMDEP), polyvinyl chloride (PVC), or chlorosulfaonated polyethylene (CSPE). They are thin prefabricated membranes produced in thickness ranging typically from 0.5 millimeters (20 mils) to 2.5 millimeters (100 mils).

2.2.2 Soil Barriers

A low hydraulic conductivity soil barrier layer without a geomembrane is shown in Figure 2.1. The physical laws governing leachate movement downward through a low hydraulic conductivity soil barrier are more complicated than those governing leachate movement through sand and gravel layers with high hydraulic conductivity. The microscopic pores that exist in clay soils causes water to move not only by an induced hydraulic head under saturated conditions but also by capillary forces under unsaturated conditions. The smaller the pore radius, the larger the capillary force. Thus, soils with high clay content will have more micro pores and large capillary forces. As the pore size of the soil increases, the capillary force decreases. Thus, silty soils have lower capillary forces than clays.

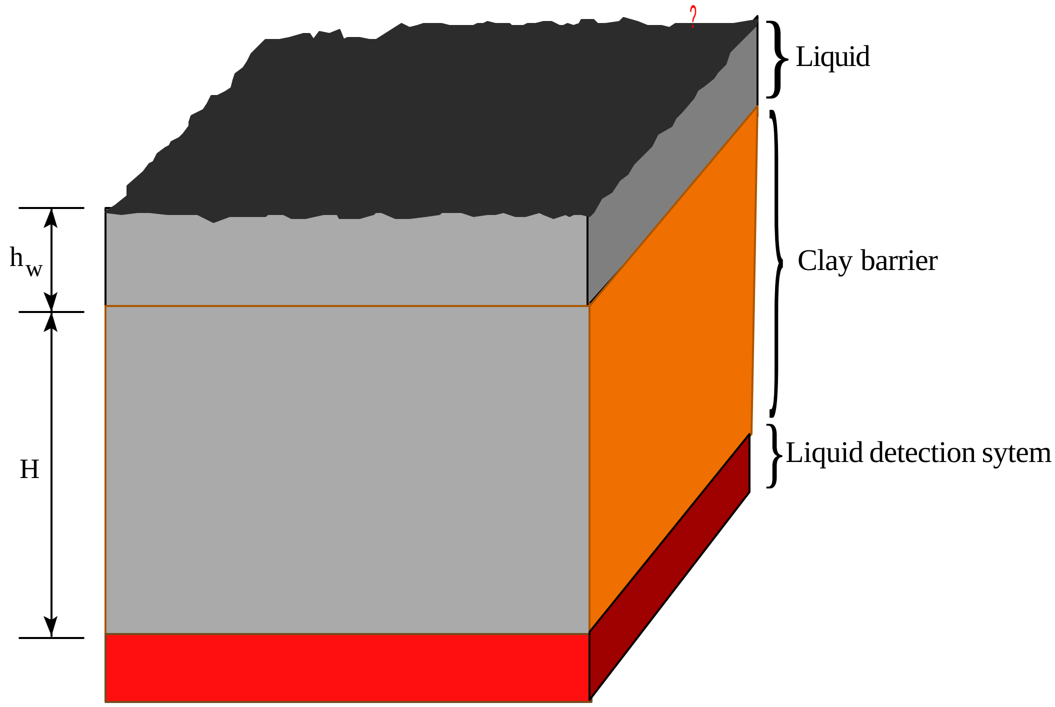


Figure 2.1 Sketch of a soil barrier system

2.2.3 Composite Barriers

A composite barrier has two elements: a geomembrane upper element and a low hydraulic conductivity soil layer as the lower element (Figure 2.2). Leakage through a composite barrier result from flow through small slits in the geomembrane and subsequently through the soil layer, or from vapor transmission through the geomembrane and again subsequently through the soil layer. In the case of leakage through a composite barrier, vapor transmission through intact geomembranes is significantly less than the rate of leakage through an imperfection in a geomembrane placed on a high hydraulic conductivity soil (Brown et al. , 1987; Jayawickrama et al. , 1988; Bonaparte et al. , 1989; Giroud and Bonaparte, 1989b; Giroud et al, 1989). Giroud and Bonaparte (1989 a) determined that the rate of leakage due to vapor transmission through an

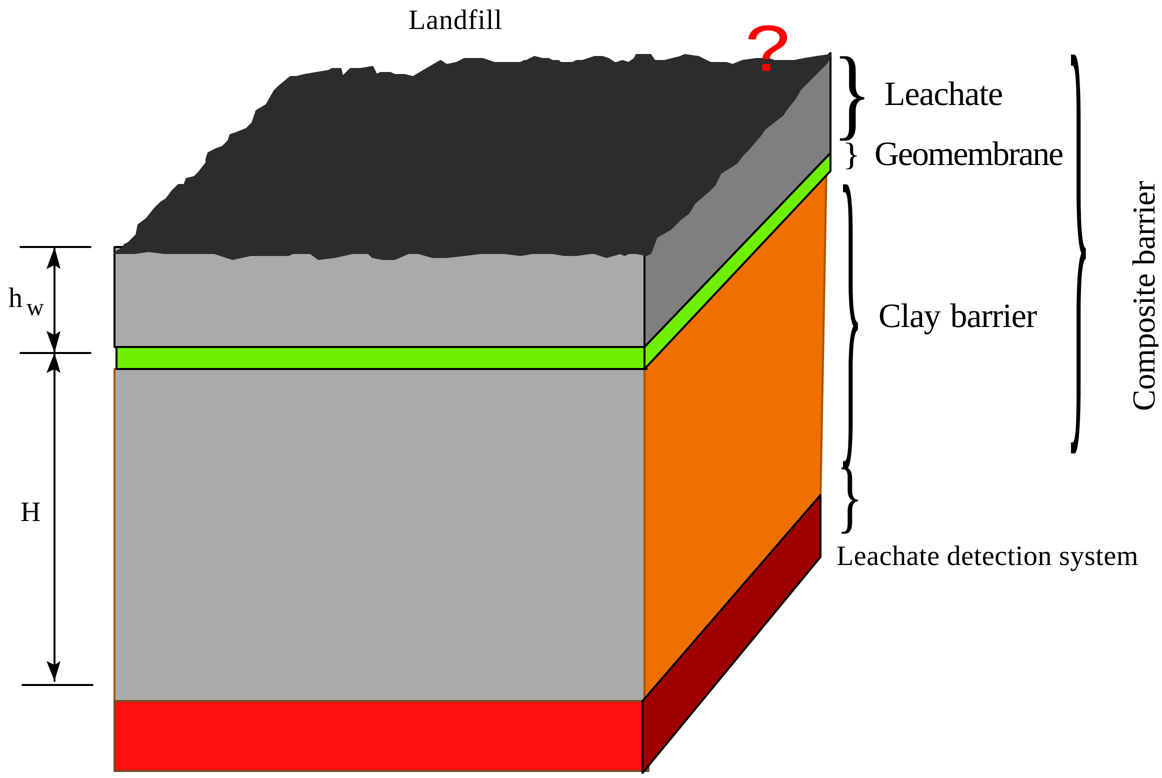


Figure 2.2 Sketch of a composite barrier system

intact geomembrane is not significantly different whether the geomembrane is placed on a low hydraulic conductivity soil or a high hydraulic conductivity soil.

To institute a method for evaluating the rate of leakage through composite barriers with slits in the geomembrane, Giroud and Bonaparte (1989a, b) reviewed the results of composite barrier model tests conducted by Sherard (1985), Fukuoka (1985), and Brown et al. (1987). They found that a key factor influencing the rate of leakage through a composite barrier was the quality of contact between the geomembrane and the soil barrier. Giroud & Bonaparte (1989b) based their findings on a combination of theoretical analyses and large-scale model tests. Brown et al. (1987) performed laboratory experiments and developed predictive equations to quantify leakage rates through various sized holes in composite barriers. They assumed a uniform, vertical percolation rate equal to the saturated hydraulic conductivity through a circular cross sectional area of the soil barrier directly beneath the slit. They developed predictive equations for the radius of this flow cross section as a function of slit size, depth of leachate pounding, and saturated hydraulic conductivity of the barrier soil. They found that the radius of saturated flow was greater than the radius of the hole in the composite barrier.

The flow of leachates through a composite barrier can be categorized into vapor transmission, flow through slits in the membrane, flow through the degraded soil component, and a combined action. These types of flow are briefly discussed below.

2.2.3.1 Vapor Transmission

In the literature, for most vapor transmission cases, steady-state saturated flow conditions are usually assumed (Shackelford, 1992), where leachate is placed on top of the geomembrane of the composite barrier. If there is a hole in the geomembrane, the leachate flows first through the geomembrane hole, then laterally some distance between the geomembrane and the low-hydraulic conductivity soil. Finally, the liquid infiltrates the soil and flows vertically down into the low hydraulic conductivity soil layer. Flow at the soil-geomembrane interface is called interface flow, and the area covered by the interface flow is called the wetted area. There is no interface flow if the geomembrane and soil are in perfect contact, however, this is an ideal case, extremely difficult to achieve in practice (Giroud and Bonaparte 1989 a, b; Fukuoka 1988; Brown 1987; Giroud et al. 1992). Other considerations affecting the rate of flow through a composite barrier are: the size of the hole, hydraulic conductivity of the soil underlying the geomembrane, and the head of leachate on top of the geomembrane.

2.2.3.2 Slits in the Geomembrane

Geomembranes are used for waterproofing hydraulic structures because of their low hydraulic conductivity. Due to their small thickness, leaks might occur easily as a result of ruptures from construction/installation, inherent imperfections or imperfect welding of seams or joints. Thus, geomembranes do not guarantee a completely impervious barrier. The hole in the geomembrane can vary in size and shape such as a slit, a punch, or a pin hole. When the hole is small, it has little influence on the leakage rate. It is reasonable to expect three to five geomembrane holes per hectare (one or two slits per acre). (Giroud and Bonaparte 1989 a).

According to Giroud et al. (1992) holes should be distinguished from pin holes, and can be defined as openings having a dimension (diameter) about as large as, or larger than, the geomembrane thickness. For holes, the flow in the low hydraulic conductivity soil layer is perpendicular to the plane of the geomembrane. Leakage rates through geomembrane holes are affected by the material underlying the geomembrane. Two extreme cases can be considered:
1) high hydraulic conductivity soil, such as a granular drainage medium, and 2) low hydraulic conductivity soil, such as a clay layer. In the case of a geomembrane hole, the rate of leakage through a composite barrier (geomembrane and a low hydraulic conductivity soil) is significantly less than the rate of leakage through a similar hole in a geomembrane placed on high hydraulic conductivity soil (Giroud et al., 1992). Equation for leakage rate for the case of small geomembrane holes can be derived by considering circular or square holes. These equations are given by Bonaparte et al. , (1989); Giroud and Bonaparte, (1989 b) and Giroud et al., (1989).

2.2.3.3 Degraded Soil Component

When a geomembrane has a rupture, the flow is governed by the low hydraulic conductivity soil underlying the geomembrane. The flow below the geomembrane will depend on hydraulic conductivity of the soil, the depth of the water table, and the boundary conditions, i. e. , permeable or impervious barrier at a specific depth. Large volumes of leakage can occur if the geomembrane is placed on a pervious medium and subjected to a large hydraulic head. The geomembrane component decreases the leakage rate, while the low hydraulic conductivity soil component increases the breakthrough time. On the other hand, while the presence of low-hydraulic conductivity soil in contact with the geomembrane decreases the rate of leakage through a hole in the geomembrane.

Some chemicals have higher rates of permeation through geomembranes than rate for water (August and Tatzky, 1984; Hoax et al. , 1984). The leachate in contact with the geomembrane may be a pure chemical, a mixture of pure chemicals, or a dilute aqueous solution. Most leachates from municipal solid waste landfills (i.e., leachates) and other nonhazardous solid waste containment facilities fall into the latter category with low concentration of a large number of chemical constituents. Leachates primarily consist of soluble, partially soluble, or miscible components removed from the concentrated leachate. This leachate can also affect hydraulic conductivity characteristics of the soil barrier (Bowders and Daniel 1987; Daniel and Liljestrand 1984). Non-water based solutions are not considered in this study.

2.2.3.4 Composite Action

A composite barrier is composed of a geomembrane and an underlying layer of low hydraulic conductivity soil placed in close contact with each other. Therefore, flow migrates first through the geomembrane and then through the soil. Leakage through a composite barrier can result from flow through geomembrane holes or vapor transmission through the geomembrane. In the case of a geomembrane hole, the rate of leakage through a composite barrier is significantly less than the rate of leakage through a similar slit in a geomembrane placed on a high hydraulic conductivity soil, as discussed by Giroud and Bonaparte (1989 a, b). The soil significantly impedes the flow of leachate through the slit, assuming that the geomembrane is in close contact with soil such as clay or clayey soil.

Generally, some lateral flow occurs between the geomembrane and the underlying soil. Giroud and Bonaparte (1989 a, b) and Giroud et al. (1992) examined the experimental and theoretical results of other researchers. They studied flows for perfect, good, and poor contact between the geomembrane and the soil. Their studies led to the following two empirical equations for the rate of leakage through a hole in the geomembrane component of a composite barrier:

For good contact conditions:

$$Q = 0.21\ a^{0.1}\ h^{0.9}\ k_s^{0.74} \tag{2.1}$$

For poor contact conditions:

$$Q = 1.15 \ a^{0.1} \ h^{0.9} \ k_s^{0.74} \tag{2.2}$$

where Q = rate of leakage through a slit in the geomembrane component of the composite barrier; a = area of the slit in the geomembrane, h = head of leachate on top of the geomembrane, and k_s = hydraulic conductivity of the low hydraulic conductivity soil component of the composite barrier. These two empirical equations are not dimensionally homogeneous and can only be used with the following units: Q (m^3/s), a (m^2), h (m) and k_s (m/s).

The use of these equations should be restricted to cases where hydraulic conductivity of the soil is less than 1×10^{-6} m/s. Also, these equations should be restricted to cases in which the head of leachate on top of the geomembrane is less than the thickness of the soil layer.

The good contact condition corresponds to a geomembrane with few wrinkles and a soil barrier, which has been sufficiently compacted, and smoothed on the upper surface. The poor contact condition corresponds to a geomembrane with wrinkles, laid over an uneven soil surface. These two contact conditions can be considered as typical field conditions.

2.3 Leaking Barriers

The rate of leakage through a composite barrier is independent of the overlying material. The flow is essentially governed by the soil underlying the geomembrane. The head loss at the geomembrane hole is negligible compared to the head loss in the underlying soil. Therefore, if the material overlying the geomembrane is more permeable than the soil of the composite barrier, no significant head loss will take place in the overlying material. Therefore, the presence of overlying material will not significantly affect the leakage rate unless fine particles migrating from the overlying material clog the geomembrane hole and the space between the geomembrane hole and underlying soil.

There is typically a leachate collection system above the geomembrane. The objectives of the leachate collection system are to accumulate the leachate that has seeped through solid waste and transport it to a sump from where it is drained and collected from the waste containment system. Thus the hazard of leakage is minimized by preventing leachate accumulation on top of the barrier.

2.4 Numerical Modeling of Unsaturated Flow

A review of the literature on numerical modeling of unsaturated flow through porous media was conducted during the development of this code. Based on this review, a model was developed to study the flow of leachate through the composite barrier and the progression of the wetting front to the underlying leachate detection system.

Mathematical modeling of the leakage through soil barriers or composite barriers has been attempted by Remson, et al. (1968), Fen, et al. (1975), Perrier and Gibson (1980), Skaggs (1980), Miller (1984), Schroeder (1984), Schroeder et al., (1984), and Paniconi et al. (1991). The first three of these researchers used the water balance method without considering the mechanism of the fluid flow through the soil profile to calculate leakage rates. This approach grossly underestimates field conditions such as a homogeneous mass of clay with uniform hydraulic properties (Gee, 1981; Gibson and Malone, 1982). Skaggs (1980) improved over the earlier attempts by incorporating Darcy's law in his model to compute the rate of moisture movement through the saturated soil profile. Miller (1984), Schroeder (1984), and Schroeder et al. (1984) used a quasi two dimensional, deterministic approach to model the saturated clay barrier. Miller's model also covers the full range of saturation. His model assumed that the surface inputs (i.e., flow rate boundary condition) have been solved for a priority using a model of surface hydrology.

For all of these studies, it was assumed that the clay barrier was homogeneous with uniform hydraulic properties. Daniel (1984) found that fluid moves much faster through actual clay barriers than predicted by using laboratory determined values of hydraulic conductivity. He found that the actual hydraulic conductivities of the clay barriers, calculated from field leakage rates, were from 5 to 10,000 times larger than those obtained from the laboratory tests on undisturbed clay samples. Daniel (1984), and Boynton and Daniel (1985) attributed the increased hydraulic conductivity in the field to the presence of the macro-pores in the clay mass.

Edwards, et al. (1979) reported that Gardner (1962) also showed that the holes and the cracks that are open at the surface can move significant volumes of the surface water much deeper and faster than that calculated by Darcy-type solutions alone. The tracer studies of Ritchie et al. (1972), Blake et al. (1973), and Quisenberry and Phillips (1976) also revealed that surface applied water flows through the interconnected macropores.

Some general features are common to many landfills and are assumed to exist in the hypothetical landfill for which the numerical model was formulated. These features are: (1) the local groundwater table is well below the landfill, and the leachate drains freely to the leachate detection system, (2) precipitation provides the only additional source of leachate to the landfill, apart from the liquid (moisture content) of the waste, and (3) the leachate detection system is designed to prevent the pounding of water within the landfill.

Most analytic solutions related to unsaturated flow through porous media appear in infiltration studies. In this study, infiltration is defined as the wetting of a vertical soil profile from the surface to lower portions. Parlange and Aylor (1972) presented analytical solutions for both the specified moisture conditions and specified flux conditions as the upper boundary. A two-parameter infiltration model for arbitrary rainfall rates was also introduced by Parlange (1975).

Skaggs (1980) presented a more rigorous approach to the barrier problem. His model considers only saturated flow through the clay barrier and does not couple the transport mechanism of the various landfill layers. Perrier and Gibson (1980) developed a computer model, Hydrologic Simulation of Solid Waste Disposal Sites (HSSWDS), to simulate transport through landfill covers based on water balance concepts.

The solution to the general flow equation for an unsaturated clay barrier yields moisture content profiles in the clay barrier at different times and also the rate of seepage flux through the barrier at different locations as a function of time. Notable among the available programs to solve the unsaturated flow equation in a hydraulic barrier system is a finite difference program, "SOILINER" developed by Goode and Smith (1986). SOILINER is a finite difference approximation of the nonlinear, governing equation for one-dimensional, unsaturated flow in the vertical dimension. SOILINER was designed to simulate the dynamic processes of an infiltration event across a compacted soil barrier system beneath impounded leachate. Since the governing equation reflects barrier heterogeneity and the dependence of barrier properties on the degree of saturation, SOILINER is capable of representing infiltration for a variety of soil (e. g. , clay) barrier scenarios. Important features inherent to the SOILINER model include the ability to simulate multilayered systems, variable initial moisture contents, and changing conditions on the boundaries of the compacted soil barrier flow domain.

Another computer model, the Hydraulic Evaluation of Landfill Performance model (HELP), simulates the effects of hydraulic processes on the water balance for landfills by performing daily sequential analyses using a quasi-two dimensional, deterministic approach (Schroeder et. al., 1987). The main hydraulic considerations include precipitation of any form, surface storage, surface evaporation, runoff, snow melt, infiltration, vegetation, plant root depth, plant transpiration, soil evaporation, soil moisture storage, soil moisture potential, unsaturated flow, and vertical and lateral saturated flow. The code handles each of these considerations, often in a simplified manner, to estimate runoff, evapotranspiration, vertical drainage to lower barriers, percolation through barriers, and lateral drainage.

Also, the two-dimensional finite element model, UNSAT2, developed by Neuman (1973) and Neuman et al. (1975), simulates moisture movement through laboratory columns and a field-scale cover design. The model uses a Galerkin type, finite element method to simulate two-dimensional, nonsteady flow of water in incompressible, saturated-unsaturated soils. Evaporation and water uptake by roots are also considered.

The unsaturated flow problem for landfill barriers is particularly hard to approach analytically because of the strong nonlinearities in the soil moisture, hydraulic conductivity and suction relationships. In addition, the initial conditions are not spatially constant and the boundary conditions may be time-dependent. Fenn et al. (1975) adopted a strategy, which is similar to the agricultural type water balance used for scheduling irrigation. The balance uses primary precipitation components to account for the precipitation that ultimately appears as leachate. Gibson and Malone (1982) applied Fenn's water balance method to a landfill test cell and reported that the predicted annual leachate accumulation is underestimated by 20%.

The volume of moisture passing through the topsoil layer represents the maximum possible volume of leakage through the soil barrier. The volume of leakage from an individual rainfall event depends upon the duration of the rainfall and the rate of leakage through the topsoil layer. This transient rate of leakage through the topsoil layer depends on antecedent moisture conditions, soil properties, soil cover, and rainfall intensity. Calculation of the rate and volume of percolation is critical since it represents the primary input to leachate generation. A numerical model based on Richard's equation uses the partial differential equation governing vertical flow in an unsaturated soil profile to simulate fluid movement in the clay and waste layer. In this study, a computer program was developed to solve Richards' equations.

Richards' equation for flow through an unsaturated soil profile (Richards 1931) is as follows:

$$\boxed{\frac{\partial}{\partial x}\left(K_x(y)\frac{\partial y}{\partial x}\right)+\frac{\partial}{\partial y}\left(Ky(y)\frac{\partial y}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z(y)\left(\frac{\partial y}{\partial z}+1\right)\right)=C(y)\frac{\partial y}{\partial t}} \tag{2.3}$$

where K_x (ψ), Ky (ψ), and K_z (ψ) are the unsaturated hydraulic conductivities (which are functions of pressure head, ψ) in the three coordinate directions x, y, z where the z axis is assumed to be vertical and θ = θ (ψ) is the volumetric moisture content of the soil barrier, and C (ψ) is the specific moisture capacity.

2.5 Finite Element Method

Finite element and finite difference methods are the most common numerical procedures for modeling groundwater problems. Finite difference methods are based on a discretization of the flow domain into a grid that is typically rectangular, where the potential or head is determined at the grid points by solving the differential equation in a finite difference form throughout the grid.

Another numerical method is the finite element method. It is more flexible than the standard form of the finite difference method. The finite element method differs from finite difference method in two ways. First, the field is discretized into a grid (mesh) of finite elements of any shape such as triangles or rectangles. Second, the differential equation is not solved immediately but replaced by several equations. For the finite element method used in groundwater flow problems, the flow domain is subdivided into small elements, for which the flow in each element is characterized in terms of the hydraulic head at the nodal points. Then, a system of equations is acquired from the barrier system so that the flow must be continuous at each node.

Analytical methods and numerical methods can be used to obtain solutions for groundwater flow. Analytical methods require a functional representation of the solution of the partial differential equation (e.g. , a mathematical expression that gives hydraulic head as a function of position and time in the soil). The primary limitation of analytical methods is that solutions can only be obtained by imposing strict limiting assumptions about soil properties, boundary conditions, or initial conditions.

Numerical methods do not need restrictive assumptions. It is possible to obtain numerical solutions for the case of anisotropic and nonhomogeneous soils and for problems with complicated and time-dependent boundary conditions. The finite element method was employed in this study.

CHAPTER 3
MATHEMATICAL MODEL

3.1 Overview

Giroud's (et al. 1989 a, b) work on leakage through the slits in geomembranes is considered as the basis for this research. In his investigation Giroud studied leakage through a single slit in the geomembrane in a composite barrier. In the present study, Giroud's results are extended to examine transient flow through an unsaturated clay barrier with geomembrane damaged by multiple slits.

The composite barrier with a geomembrane slit and the moving wetting front is presented in Figures 3.1 and 3.2, respectively. The liquid on top of the clay barrier enters into the boundary conditions of the problem. If there is no geomembrane, the boundary condition at the top of the soil profile is $\Psi = h_w$ for all point on the surface. The analyses is carried out using finite element method which takes into account barriers with different type, size and number of slits in the geomembrane.

3.2 Mathematical Model of Flow Through Geomembrane Slits

To be able to calculate the leakage and the progressive movement of the wetting front through composite barrier, it is necessary to understand the behavior of the flow through the geomembrane slit. The results of Giroud et al. (1989 a, b) give us the variable boundary condition on top of the geomembrane as shown in Figure 3.2. The presence of more than one slit in the geomembrane and the resulting overlap of the wetting fronts is the main focus of this work. The initial conditions necessary to analyze flow through the composite barrier can be obtained from the conditions under which the composite barrier was constructed.

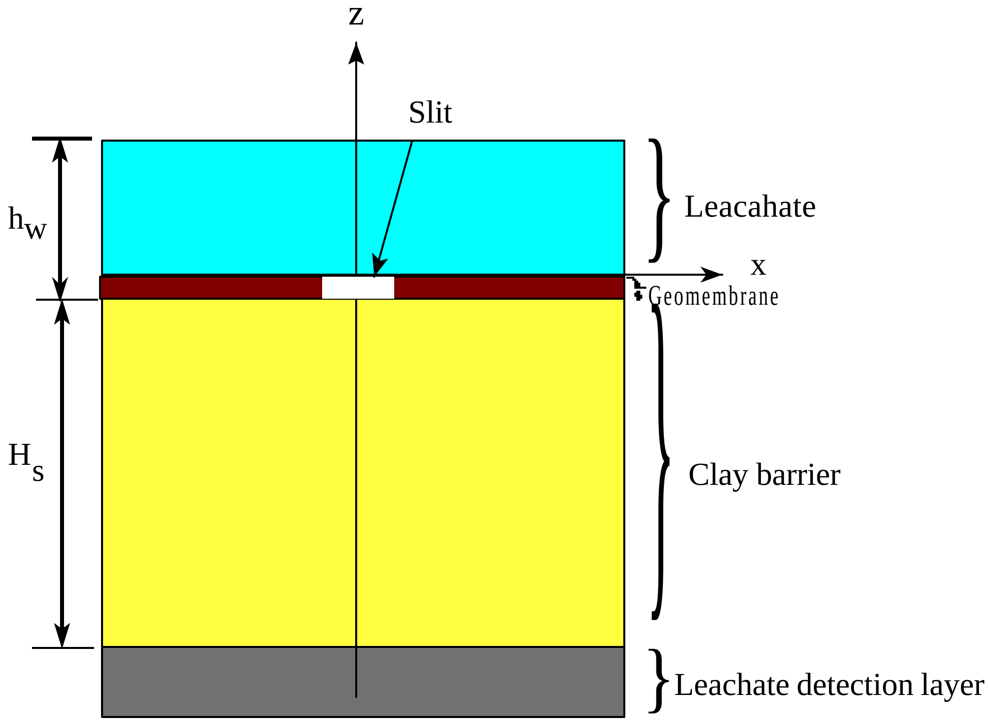


Figure 3.1 Composite barrier with a damaged geomembrane

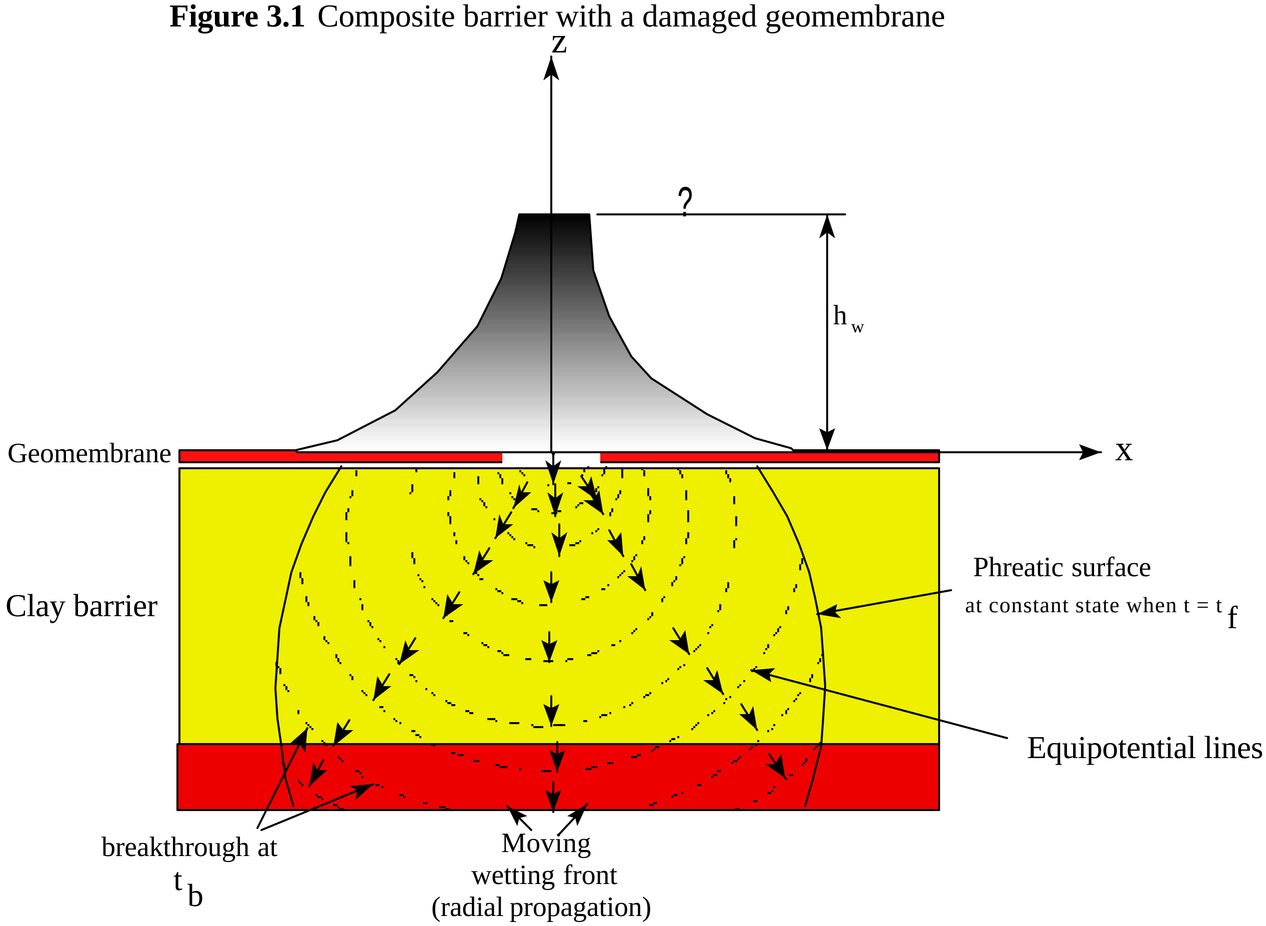


Figure 3.2 Moving wetting front for the composite barrier with a damaged geomembrane

According to experimental studies, (See Giroud and Bonaparte 1989) the head of liquid on top of the soil corresponding to a single slit decreases progressively from a maximum value (h_w) over the slit itself to zero at the edge of the wetting area (Figure 3.3 a). The top of the soil barrier is a reference level for the pressure head. The origin of the x-axis, perpendicular to the slit, is on the soil surface at the center of the slit. R is the width of the wetting area, R_0 is the width of the slit in geomembrane, and h_w is pressure head on top of the soil layer (Figure 3.3 b). The relationship between the head of liquid h(x) and the distance (x) form the center of the slit is given by Equation 3.1.

$$h(x) = \begin{cases} h_w & \text{if} \quad |x| \le R_0 \\ h_w \dfrac{\left[\ln\left(\dfrac{R}{x}\right)\right]}{\ln\left(\dfrac{R}{R_0}\right)} & \text{if} \quad |x| \ge R_0 \end{cases} \tag{3.1}$$

The empirical width of wetting area obtained by Giroud and Bonaparte (1989 a, b) for various slits and contact conditions is as follows:

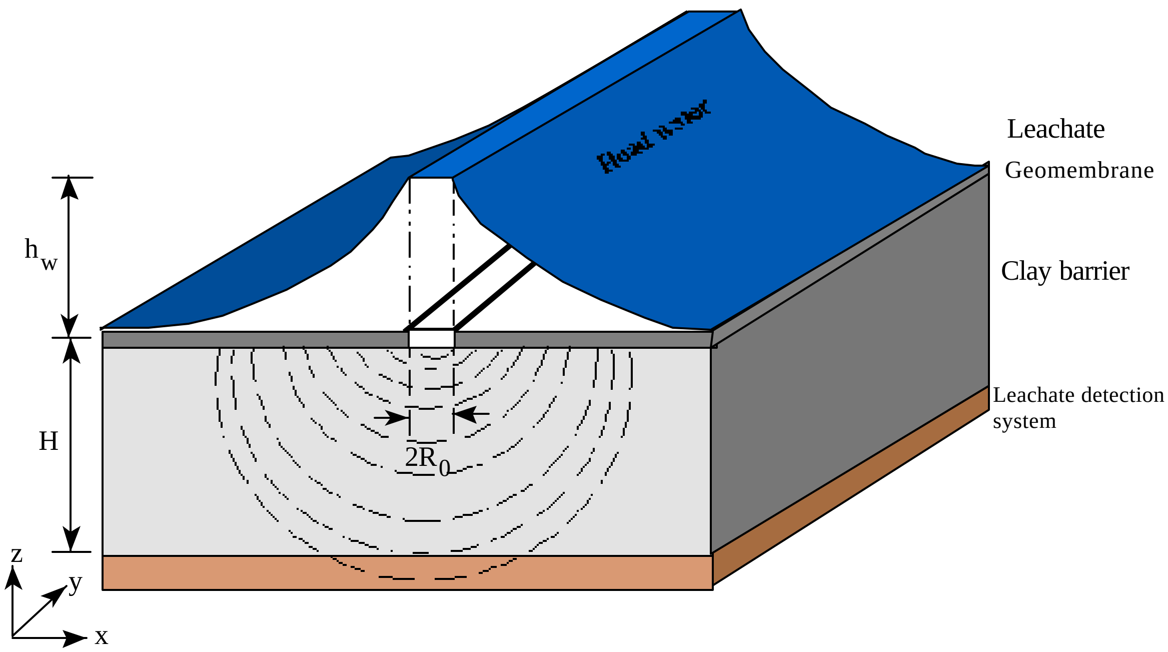
1. Poor contact:

$$R = 0.61\ R_0^{\ 0.1}\ h_w^{\ 0.45}\ K_0^{\ -0.13} \tag{3.2}$$

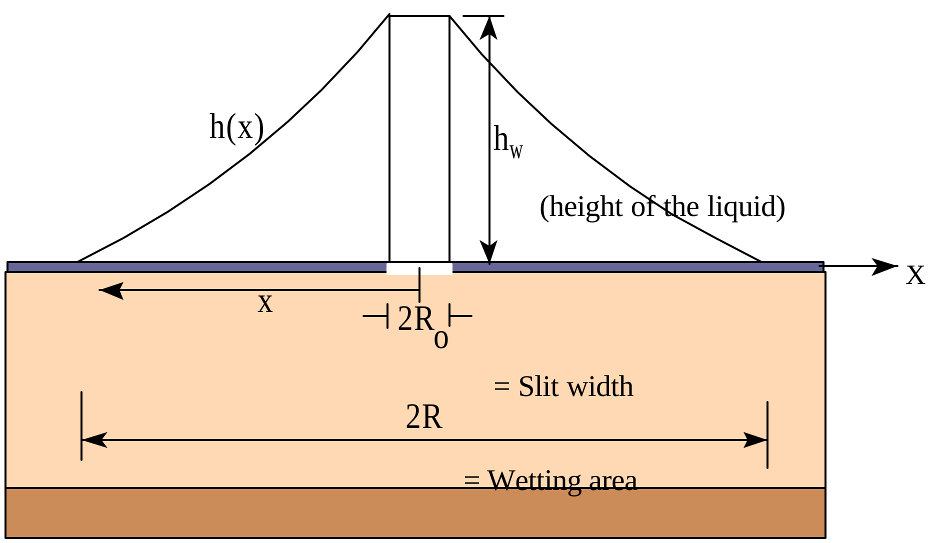
2. Good contact:

$$R = 0.26\ R_0^{\ 0.1}\ h_w^{\ 0.45}\ K_0^{\ -0.13} \tag{3.3}$$

where K_0 is the hydraulic conductivity of the saturated soil.



a) Giroud's analysis



b) 2-D cross-section

Figure 3.3 Distribution of leachate head on top of the soil

We analyze a nonstationary flow through a soil profile shown in Figure 3.4.

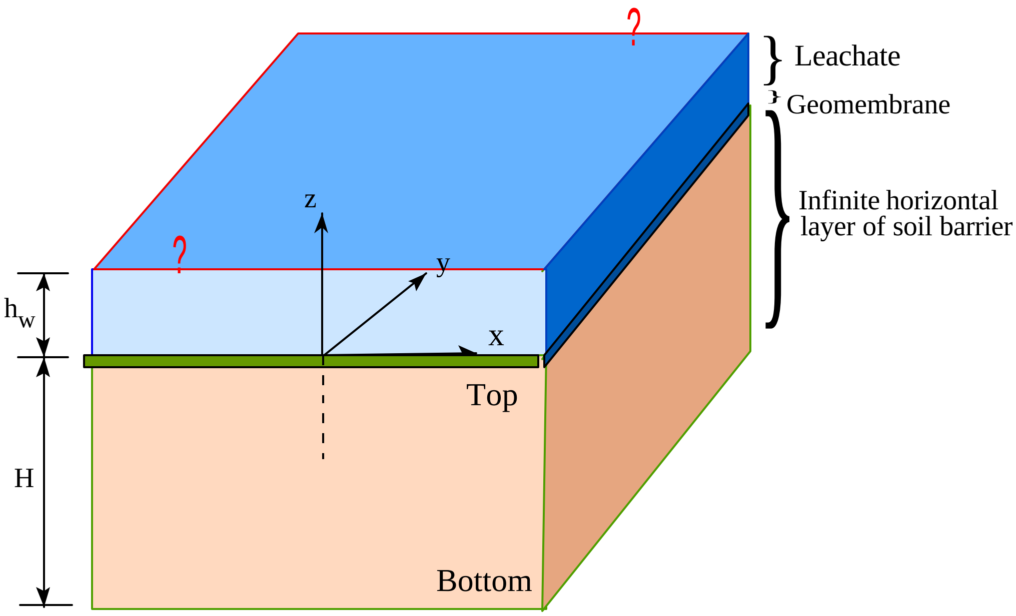


Figure 3.4 Infinite horizontal composite barrier

The Richards equation governing the flow through an unsaturated soil is,

$$\frac{\partial}{\partial x}\left(K(\psi)\,\frac{\partial \psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K(\psi)\,\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial z}\left(K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right)=C(\psi)\,\frac{\partial \psi}{\partial t} \qquad (3.4)$$

where ψ is the pressure head, K(ψ) is the unsaturated hydraulic conductivity and C(ψ) is the specific moisture capacity. The qualitative behavior of K(ψ) is shown in Figure 3.5.

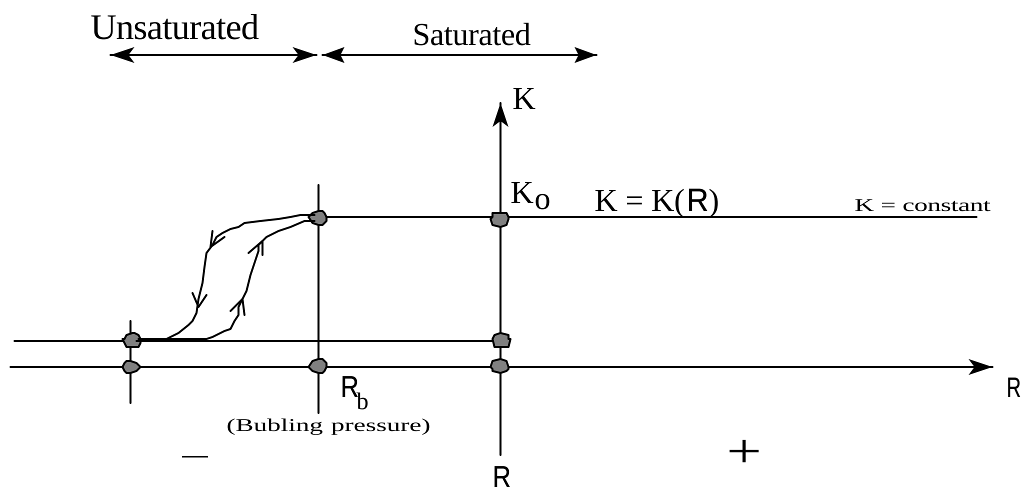


Figure 3.5 Hydraulic conductivity (K) versus pressure head (R) (Freeze and Cherry 1979)

The volumetric moisture content, $\theta(\psi)$ is defined as the volume of the water divided by the volume of at the containing it soil V_w / V_s. A typical hysteretic relationship between θ and ψ for a sandy soil is shown in Figure 3.6. The slope of this characteristic curve represents the specific moisture capacity and is defined as $C(\psi)= d\theta /d\psi$. For fine-grained soils, pressure head ψ is slightly larger than the air entry pressure head, ψ_a. The corresponding pressure, p_a, is called the air entry pressure or the bubbling pressure. As an example, a typical plot of the soil-retention curve based on Mualem's model is given in Figure 3.7.

In the saturated soil the hydraulic conductivity, K and porosity θ are constant parameters. In the unsaturated zone these parameters become function of the ψ. The qualitative dependence of K on θ is shown in Figure 3.8.

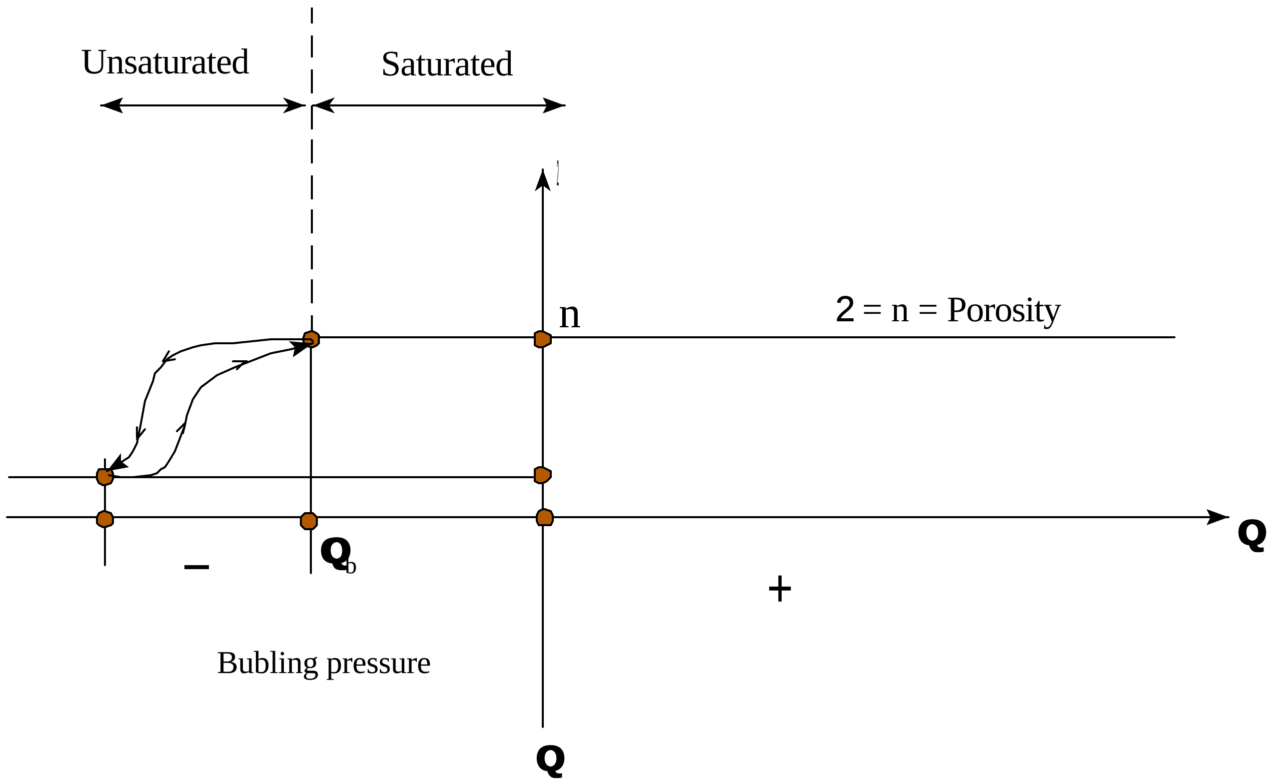


Figure 3.6 Characteristic curves of volumetric moisture content (2) versus pressure head (R) (Freeze and Cherry 1979)

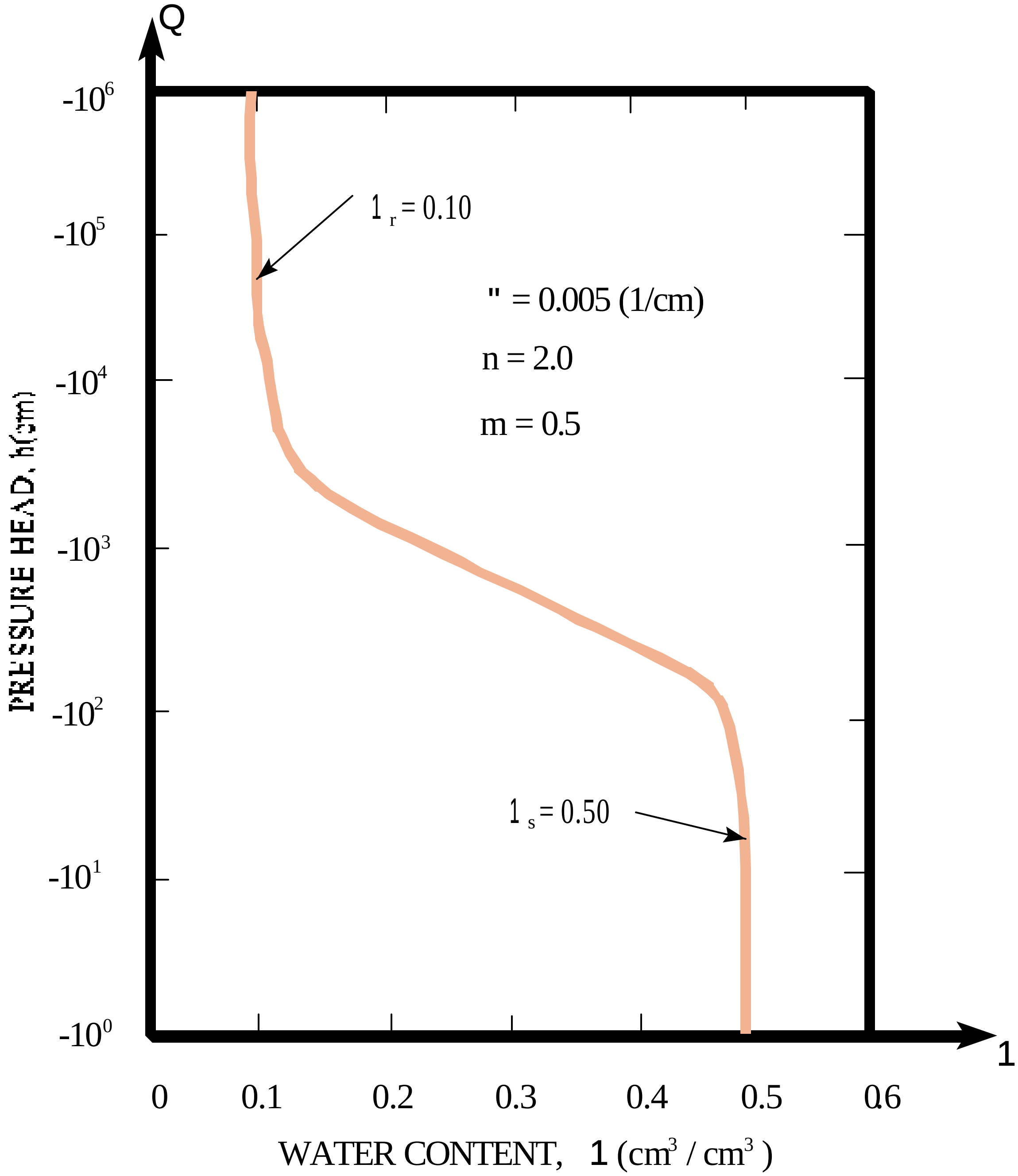


Figure 3.7 Typical plot of the soil-retention curve based on Mualem's model (Mualem, 1976)

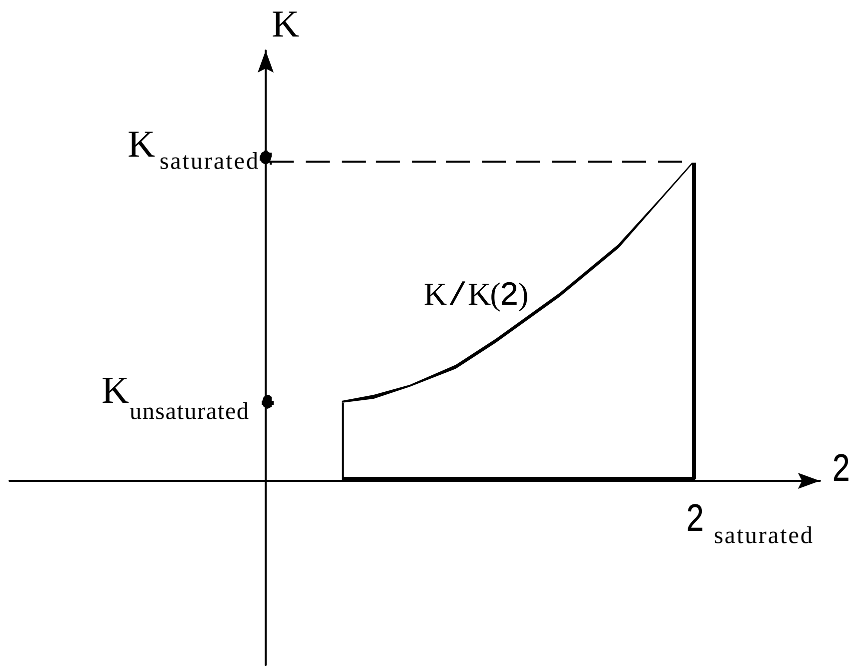


Figure 3.8 Relationship between hydraulic conductivity (K) and volumetric moisture content (2) (Philip 1969)

The Richards' equation (Richards 1931) represents the principle of conservation of mass for the fluid flow in a porous medium. The functions K(ψ) and θ(ψ) are determined empirically for each kind of soil. In this work, van Genuchten's (1980) equations are used to determine K(ψ) and θ(ψ). Based on the van Genuchten model (Mualem, 1976),

$$\boxed{K\ (y) = K_0 \frac{1-(a\,y)^{n-2}\left[1+(a\,y)^{n}\right]^{-m}}{\left[1+(a\,y)^{\,n}\right]^{2m}}} \qquad (3.5)$$

$$q(y)=q_x+\frac{(q_s-q_r)}{\left[1+(a\,y)^n\right]}^{m} \tag{3.6}$$

$$C(y)=\frac{dq}{dy}(y)=-m\left[\frac{1}{1+(a\,y)^n}\right]^{m-1}\frac{-na\,(a\,y)^{n-1}a}{\left[1+(a\,y)^n\right]^2} \tag{3.7}$$

where θ_r and θ_s are the residual and the saturated values of the volumetric water content, α and n are the shape parameters related to the air entry pressure p_a and the pore size distribution of the soil, and

$$m=1-\frac{1}{n} \tag{3.8}$$

The values of the above parameters are presented Table 3.1 (Van Genuchten, 1980).

Table 3.1 Physical properties of the different types of soil (van Genuchten, 1980).

Soil Name	θ_s (cm^3/cm^3)	θ_r (cm^3/cm^3)	K_s (cm/day)	α (cm^{-1})	n
Hygiene Sandstone	0.25	0.153	108	0.0079	10.4
Touchet Silt Loam	0.469	0.19	303	0.005	7.09
Silt Loam	0.396	0.131	4.96	2.00e-04	2.06
Gualph Loam (drying)	0.52	0.218	31.6	0.0115	2.03
Gualph Loam (wetting)	0.434	0.218	---	0.02	2.76
Belt Netofa Clay	0.446	0	0.082	0.00050000000	1.17

3.3 Overlap of Wetting Fronts: Boundary and Initial Conditions

If a geomembrane (with slits) rests on the top of the soil barrier, pressure head can no longer be determined from Equation 3.1 alone. An approximation must be made must to account for overlapping pressure heads corresponding to neighboring slits in the geomembrane (Figure 3.9 and 3.10). According to Giroud (1992) the wetting areas do not overlap if the distance between the slits is greater than 2R. If the wetting areas do overlap the pressure head from equations (3.2) and (3.3) must be added together for all points that experience the overlap. It is assumed that the geomembrane had several slits. A subroutine was written to obtain the curve h(x) (Equation 3.1) for every slit and to add the values of the pressure head at each point X. The resultant pressure head h(x) forms a boundary condition for Richards' equation for z=0 and depends on the relative position of the slits in geomembrane.

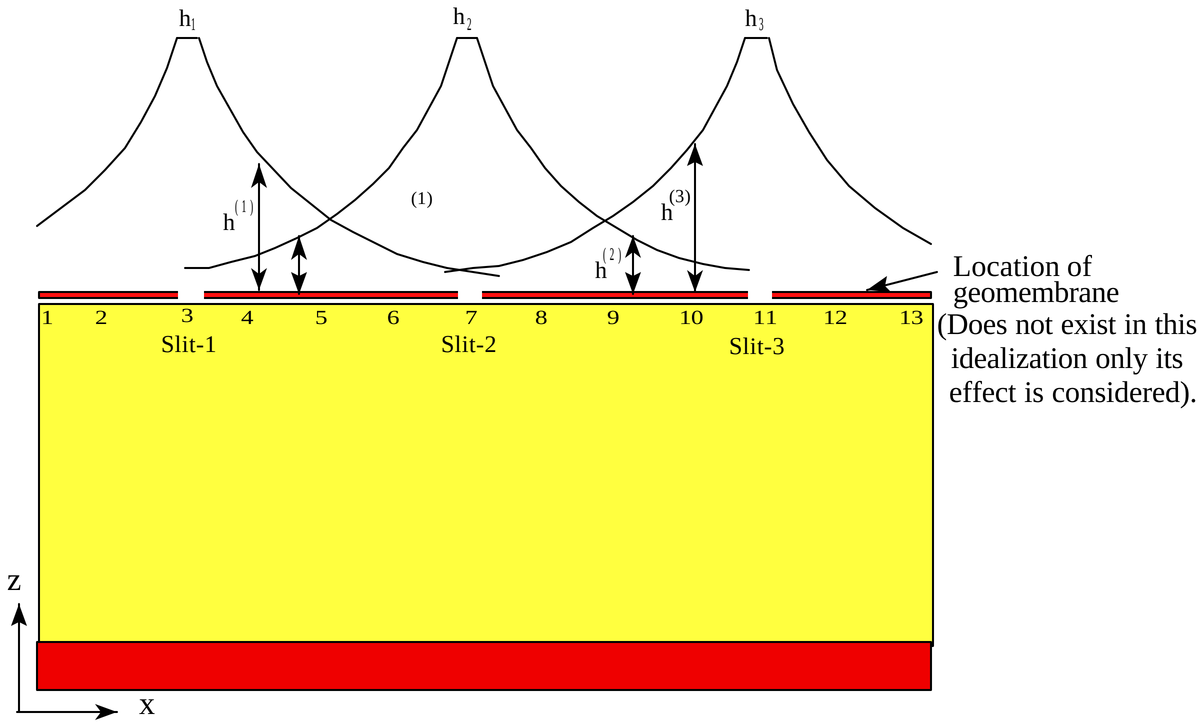


Figure 3.9 Conribution of the several holes to the pressure head (R) distribution

For the problem of overlapping slits the Richards' equation has the form

$$\frac{\partial}{\partial x}\left(K(y)\ \frac{\partial y}{\partial x} \right) + \frac{\partial}{\partial z}\left(K(y)\left(\frac{\partial y}{\partial z} + 1 \right) \right) = C(y)\ \frac{\partial y}{\partial t}, \tag{3.9}$$

where R (x, z, t) is a function of x, z and t alone. The boundary and initial conditions for Equation (3.9) are as follows.

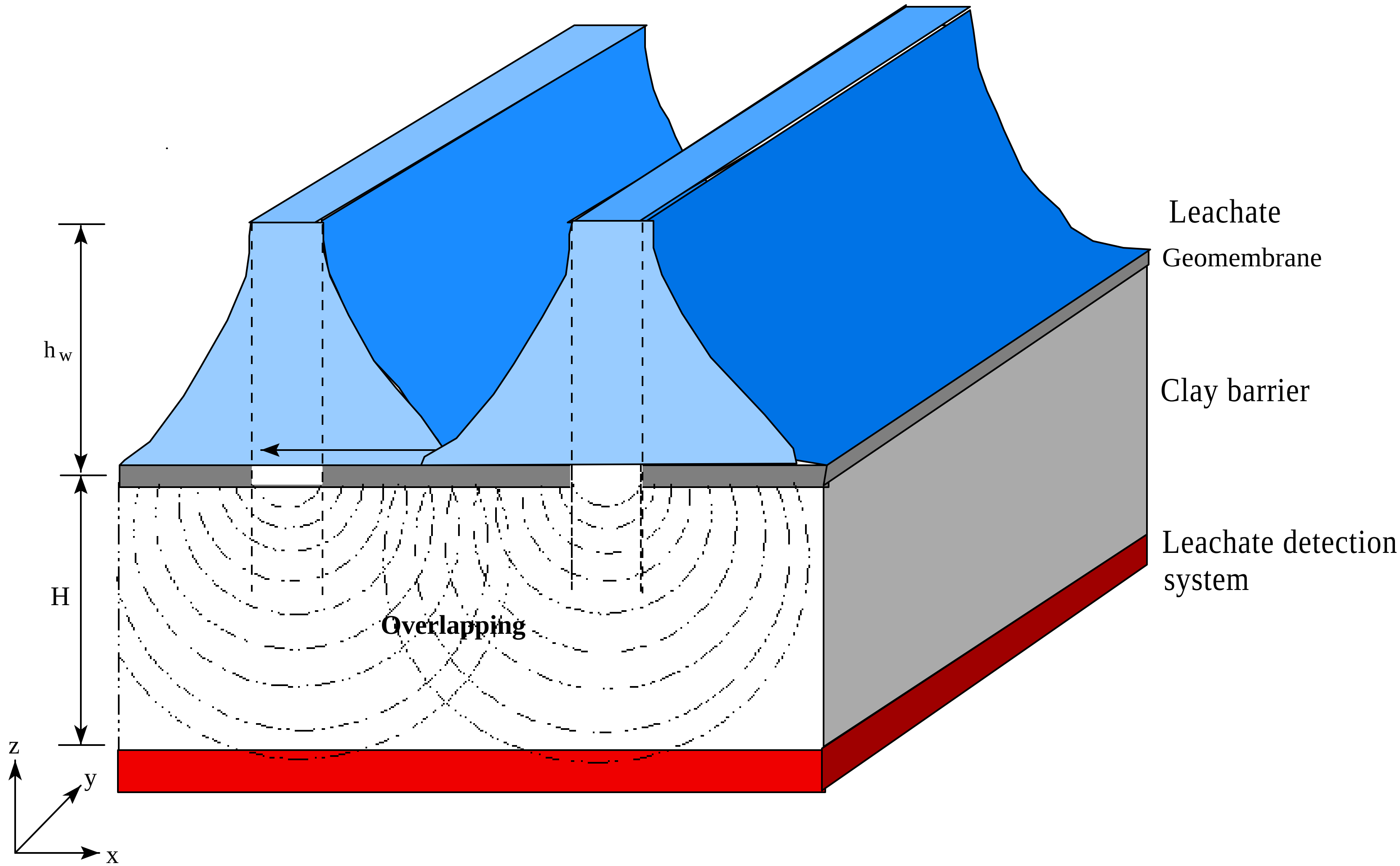


Figure 3.10 Overlap of the wetting fronts for geomembrane with several slits

1. Clay Liner with a Sand Layer

Leachate with height h_w is above the clay liner which overlays the sand layer as shown on (Figure 3.11). In this case the boundary and the initial conditions are:

$R\,(x,\ 0,\ t) \equiv h_w$ for all $t > 0$ and $z = 0$,

$R\,(x,\ -H,\ t) \equiv -C$ for all $t > 0$ and $z = -H$,

$R\,(x,\ z,\ 0) \equiv -C$ for all $t = 0$ (initial condition).

The value of C was assumed to be C=75 cm.

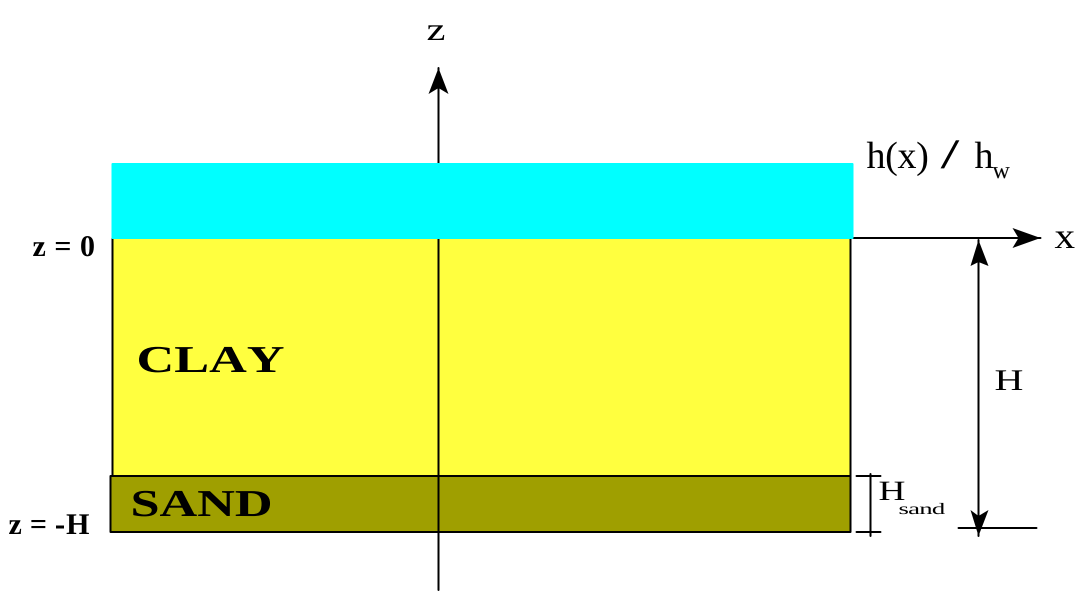


Figure 3.11 Clay liner with sand layer

2. Geomembrane Below the Sand Layer

If the geomembrane with no defect is placed below the sand layer then the boundary conditions are (Figure 3.12) :

$R\ (x,\ 0,\ t) = h_w$ for all $t > 0$ and $z = 0$,

$q = K\ (\partial R\ /\ \partial z + 1)$ for all $t > 0$ and $z = -H$,

and the initial condition is:

$R\ (x,\ z,\ 0) = -C = -75$ cm.

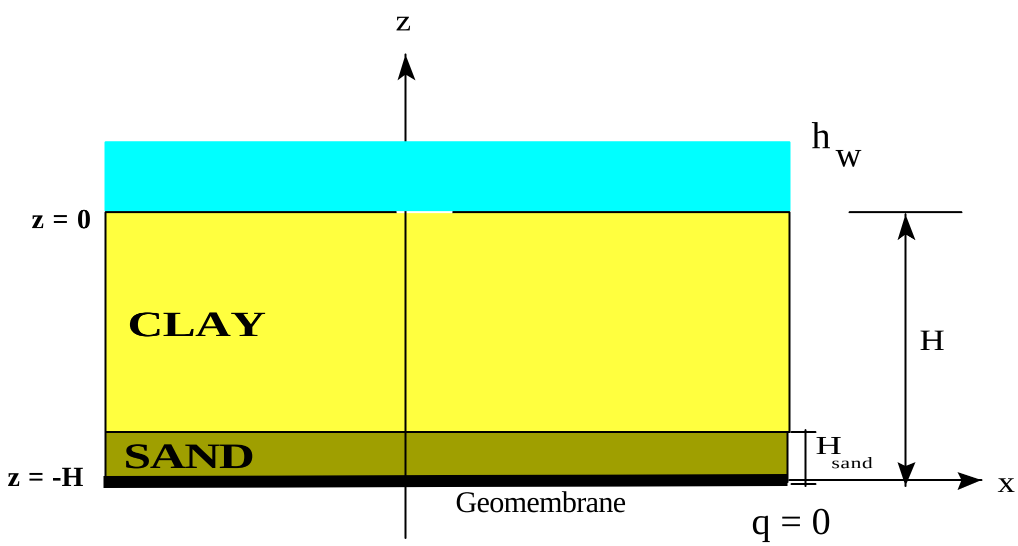


Figure 3.12 Geomembrane below the sand layer

3. Groundwater Table at the Bottom of the Sand Layer

If the groundwater table is placed at the bottom of the sand layer then (Figure 3.13), the boundary conditions are:

$\mathrm{R}(x,\ 0,\ t) = h(x)$ for all $t > 0$ and $z = 0$,

$\mathrm{R}(x,\ -H,\ t) = 0$ for all $t > 0$ and $z = -H$,

and the initial condition is:

$\mathrm{R}(x,\ z,\ 0) = -C = -75$ cm.

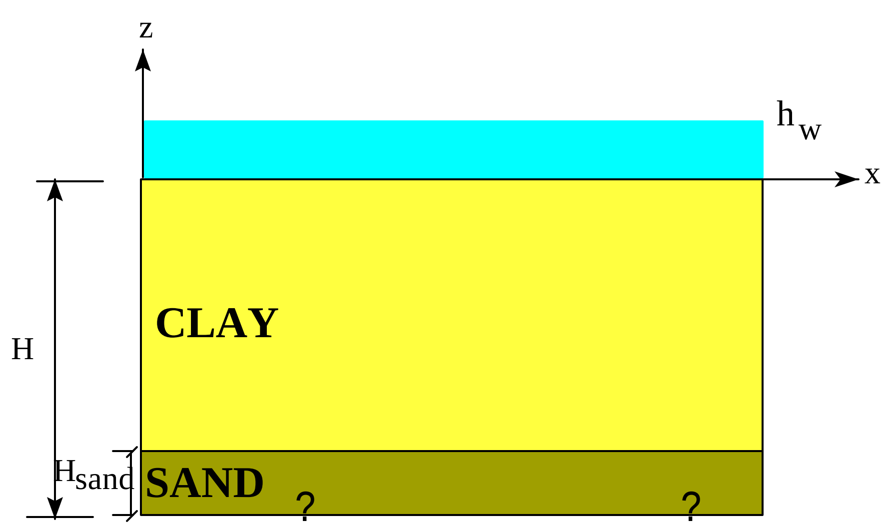


Figure 3.13 Groundwater table at the bottom of the sand layer

4. Groundwater Table within the Clay Barrier

If the groundwater table is within clay barrier then (Figure 3.14), the boundary conditions are:

$R\,(x,\ 0,\ t) \equiv h(x)$ for all $t > 0$ and $z = 0$,

$R\,(x,\ -H_w,\ t) \equiv C$ for all $t > 0$ and $z = -H_w$,

and the initial condition is:

$R\,(x,\ z,\ 0) \equiv -C = 75$ cm.

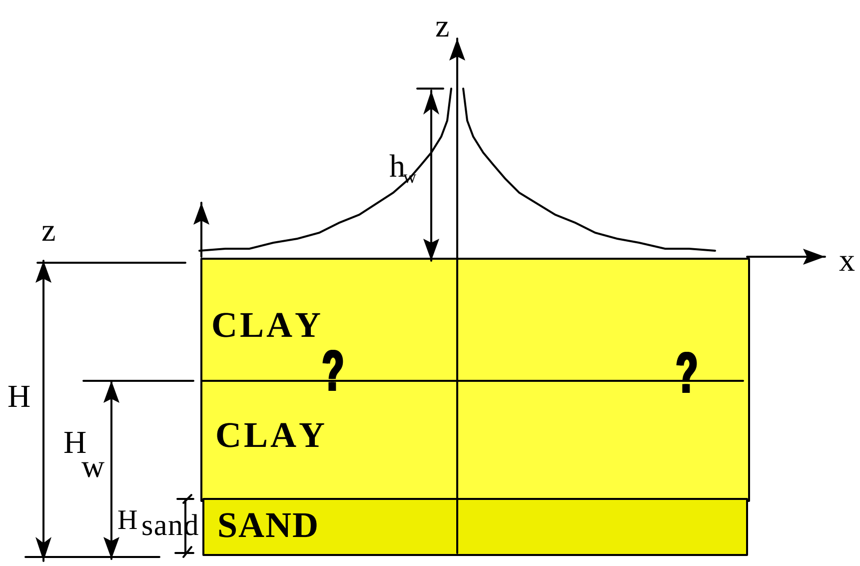


Figure 3.14 Groundwater table within the clay barrier

The appropriate hydraulic conductivity and specific moisture capacity parameters are chosen according to Van Genunchten model.

3.4 Assembly of Computer Code

In this section, a description of the computer program is presented. The main elements of this work that must be coupled in order to develop the numerical procedure are Richards' equation and the results of Giroud's experimental work. The parabolic partial differential equation characterizing fluid flow in unsaturated porous media, is generated by Darcy's law and the continuity equation (Philip, 1969). The two-dimensional form of the equation for groundwater flow through unsaturated soil is described by Equation 3.9 in Section 3.3.

The solution of this equation is carried out by the finite element method. Mathematical details of the finite element formulation can be found in the literature (Bear, 1979; Istok, 1989). The scheme used to descritize the problem is presented in Figure 3.15. A summarized flow chart of the COMPBAR computer program is presented in Figure 3.16. A listing of the computer program is given in Appendix A.

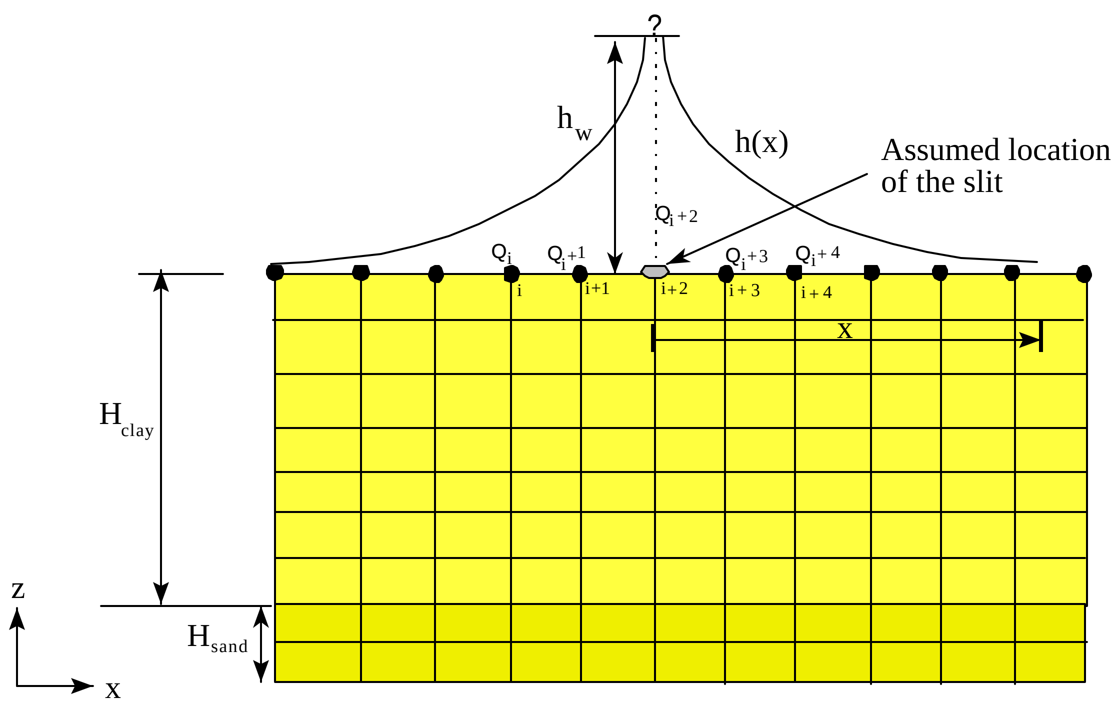
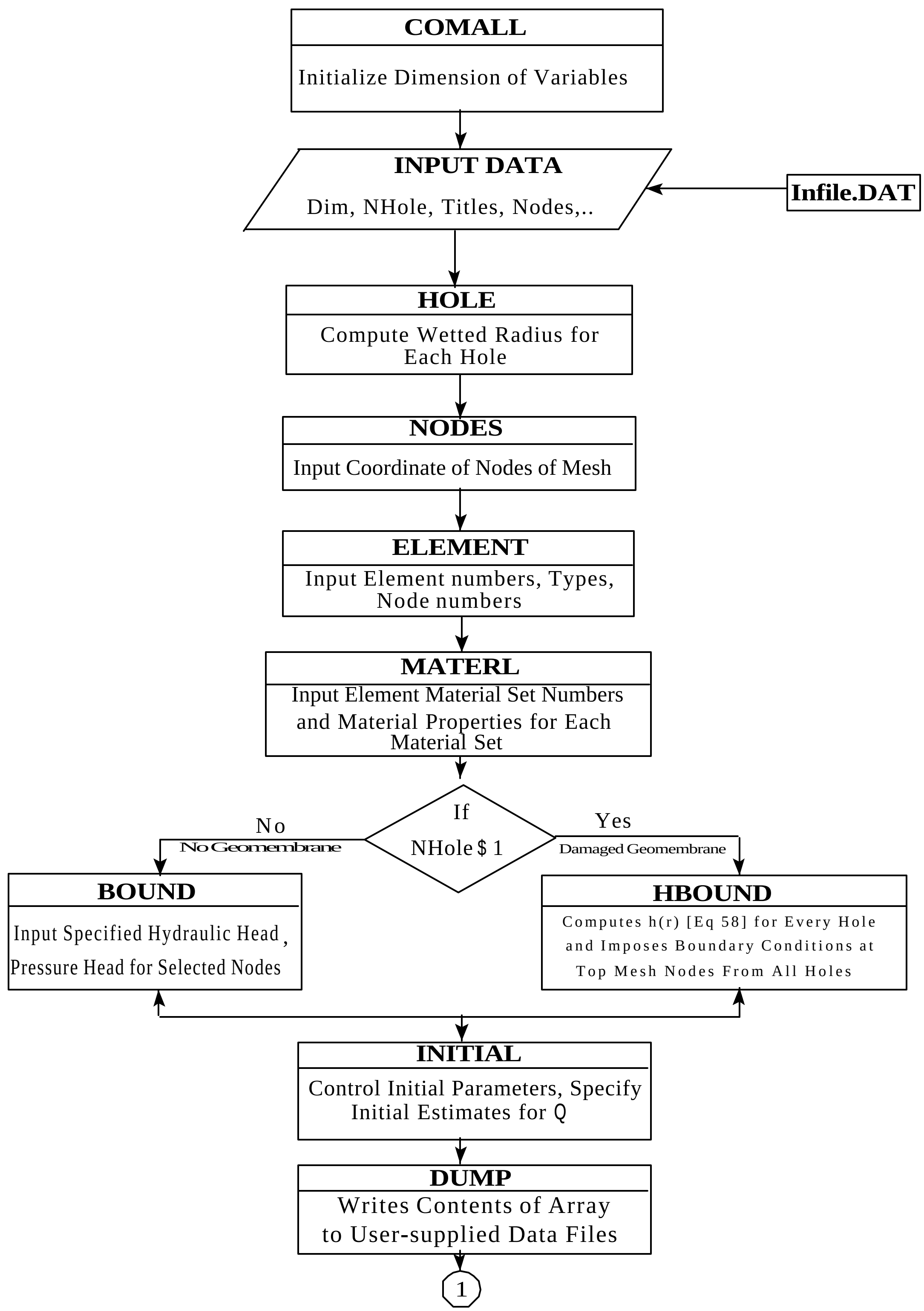
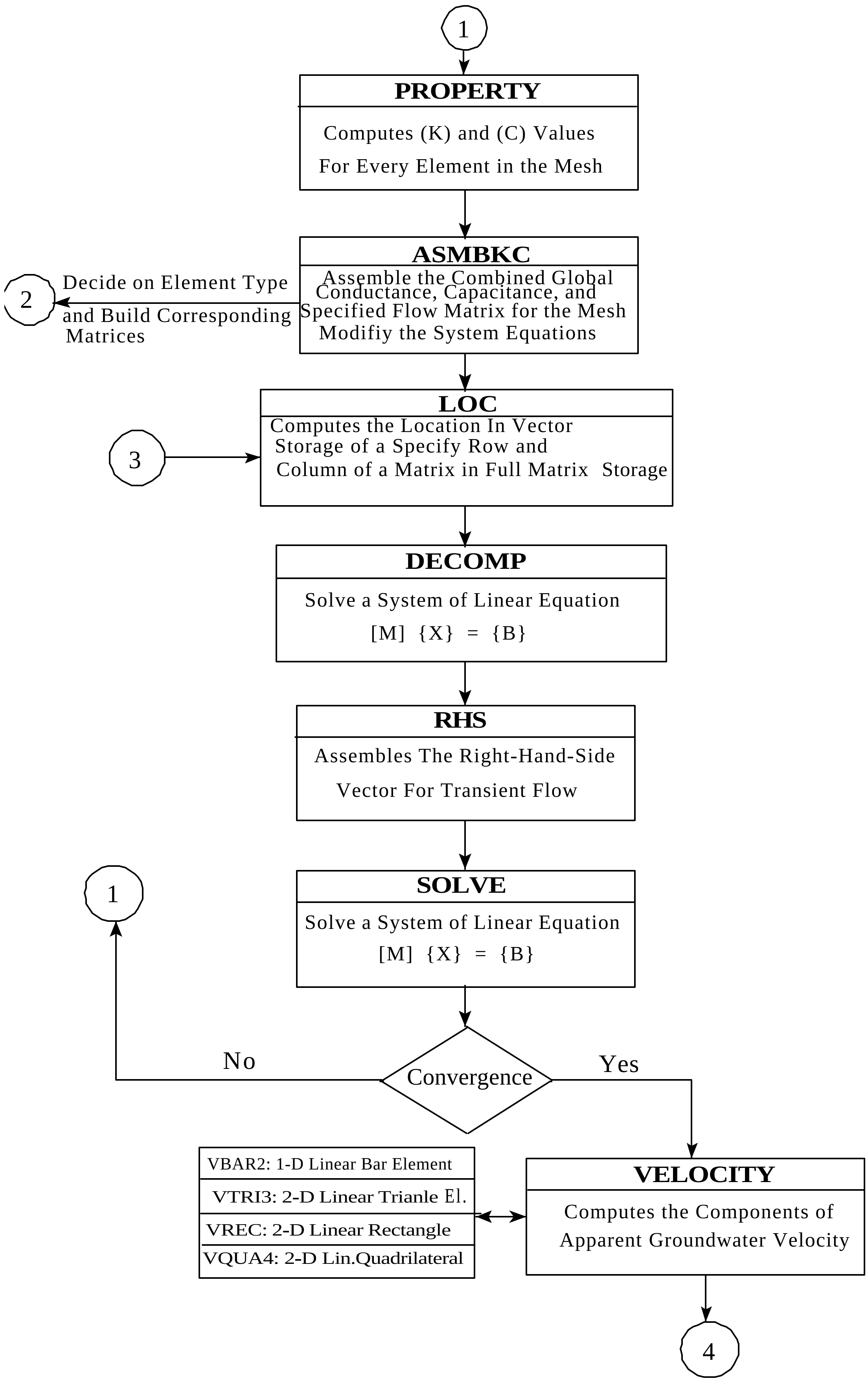
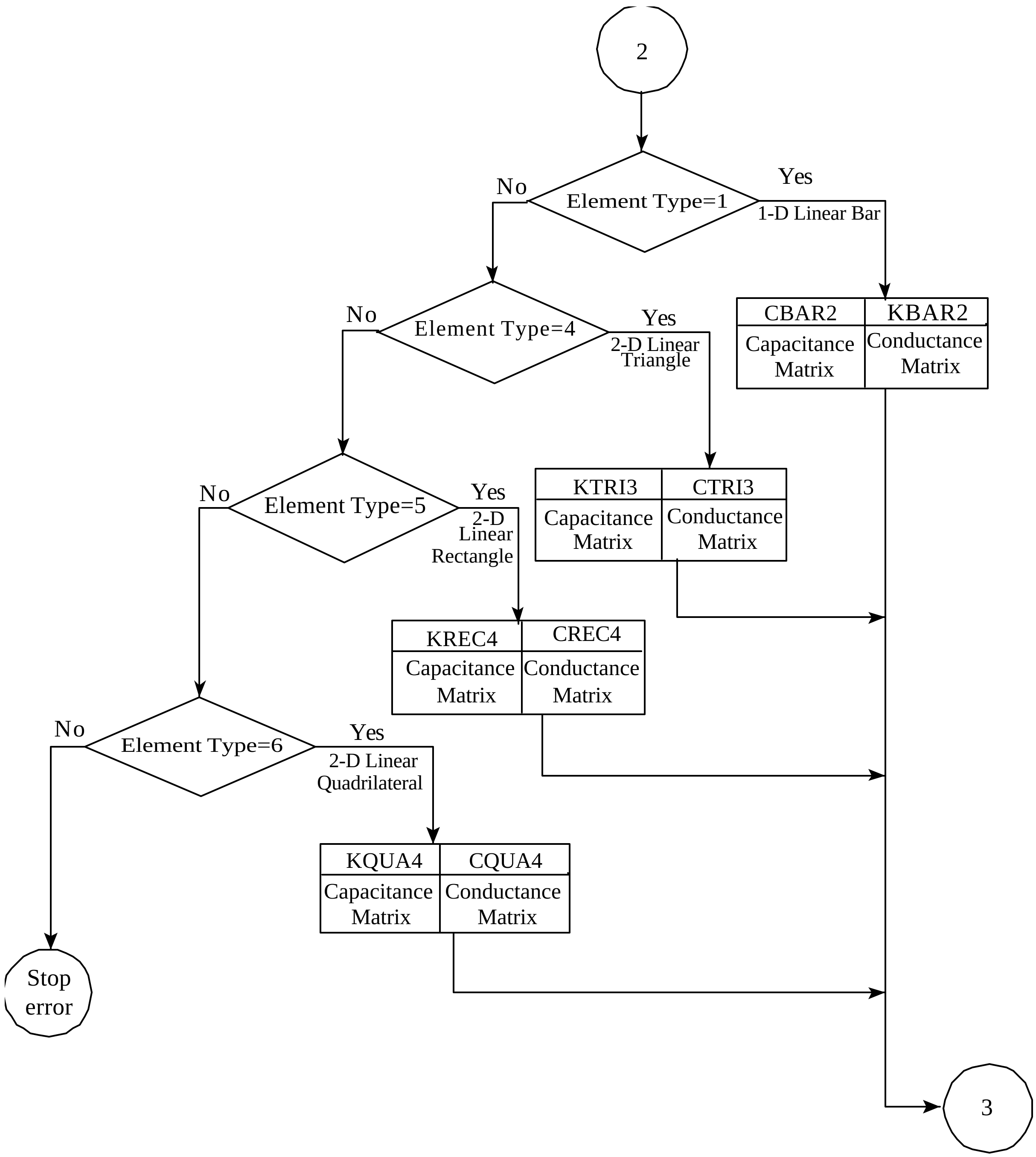


Figure 3.15 Finite element discrezation of the flow through a geomembrane slit

Program COMPBAR




1
PROPERTY
Computes (K) and (C) Values
For Every Element in the Mesh
ASMBKC
Assemble the Combined Global
Conductance, Capacitance, and
Specified Flow Matrix for the Mesh
Modifiy the System Equations
2
Decide on Element Type
and Build Corresponding
Matrices
LOC
Computes the Location In Vector
Storage of a Specify Row and
Column of a Matrix in Full Matrix Storage
3
DECOMP
Solve a System of Linear Equation
[M] {X} = {B}
RHS
Assembles The Right-Hand-Side
Vector For Transient Flow
SOLVE
Solve a System of Linear Equation
[M] {X} = {B}
1
No
Convergence
Yes
VBAR2: 1-D Linear Bar Element
VTRI3: 2-D Linear Trianle El.
VREC: 2-D Linear Rectangle
VQUA4: 2-D Lin.Quadrilateral
VELOCITY
Computes the Components of
Apparent Groundwater Velocity
4



2
Element Type=1
No
Yes
1-D Linear Bar
CBAR2
KBAR2
Capacitance Matrix
Conductance Matrix
Element Type=4
No
Yes
2-D Linear Triangle
KTRI3
CTRI3
Capacitance Matrix
Conductance Matrix
Element Type=5
No
Yes
2-D Linear Rectangle
KREC4
CREC4
Capacitance Matrix
Conductance Matrix
Element Type=6
No
Yes
2-D Linear Quadrilateral
KQUA4
CQUA4
Capacitance Matrix
Conductance Matrix
Stop error
3

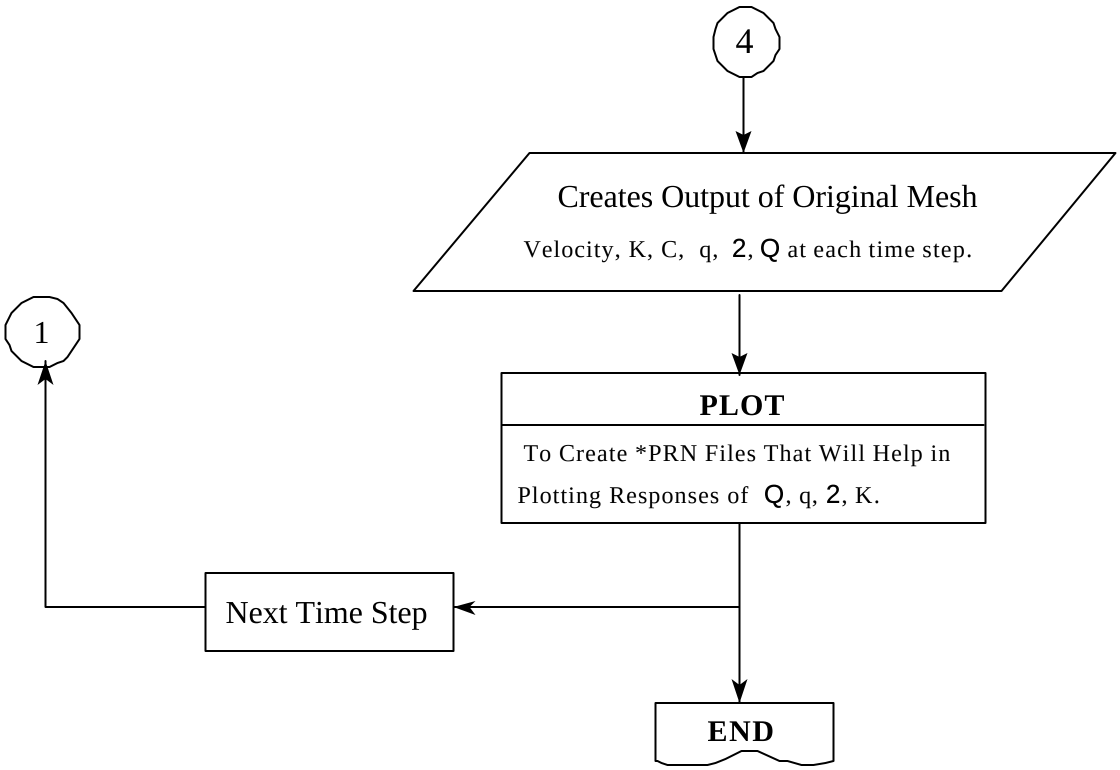


Figure 3.16 Flow chart of COMPBAR computer program

3.5 Evaluation of the Accuracy of the Code

As a verification for the model developed in this work results from the numerical programs SOILINER and SEEP are used. Giroud et al. (1989 a, b) established two empirical equations for the calculation of the flow rate as given in Equation 2.1 and 2.2 in Chapter 2. Once the flow rate (Q) is computed for a soil profile using these equations, then the time required for the liquid to migrate through a certain depth can be predicted as the follows:

$$Q = k\,i\,A, \quad v = Q/A, \quad t = s/v, \quad \text{and} \quad A = B \times 1 = 30 \times 1 = 30\ m^2,$$

where k is the hydraulic conductivity, i is the hydraulic gradient, A is the area of the cross section of flow, B is the width of the cross section of flow, v is the Darcy velocity, t is time, s is the thickness of the soil profile. The calculation of the overall flow rate (Q) from the empirical equations and the exact flow rate through the depth determined by COMPBAR is given in Table 4.9 in Chapter 4.

3.5.1 SOILINER Hypothetical Case

To evaluate the accuracy of the COMPBAR program, the same example as presented in the SOILINER manual (Example 1) was calculated using COMPBAR. A sketch for this example is presented in Figure 3.17. The comparison of both calculations is presented in Chapter 4.

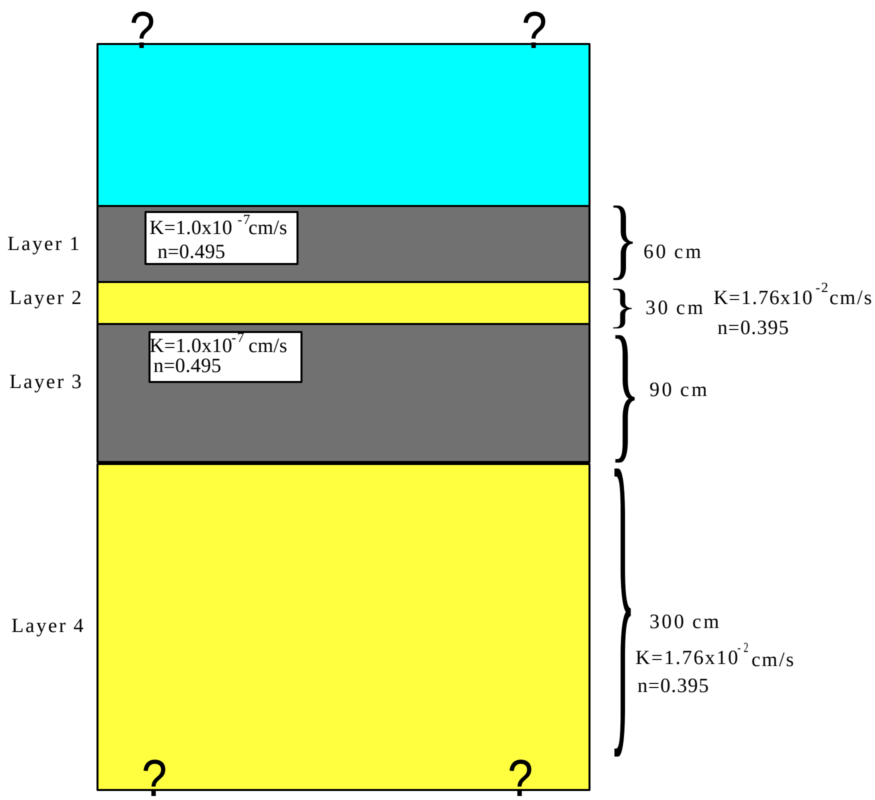


Figure 3.17 A two liner systems as simulated with SOILINER and COMPBAR (after Goode, 1986)

3.6 Parametric Analyses

Twenty different cases of a composite barrier (Figure 3.18) with a damaged geomembrane were considered. In all of these examples the initial volumetric moisture content (θ_i) in the soil barrier was assumed constant and the value of the pressure head (ψ) at top of the geomembrane was taken as –75 cm. The following assumptions were also made:

1) The geomembrane is 1 mm (40 mils) thick, with one or five slits having a width (R_0) of 1.0 to 4.5 cm.

2) The average liquid depth at the top of the geomembrane is 30 cm. The liquid is assumed to be water.

3) The thickness (H) of the clay barrier beneath the geomembrane is 90 cm and its hydraulic conductivity is $K_s = 1.0 \times 10^{-7}$ cm/sec.

4) The liquid detection system is constructed from a high hydraulic conductivity material.

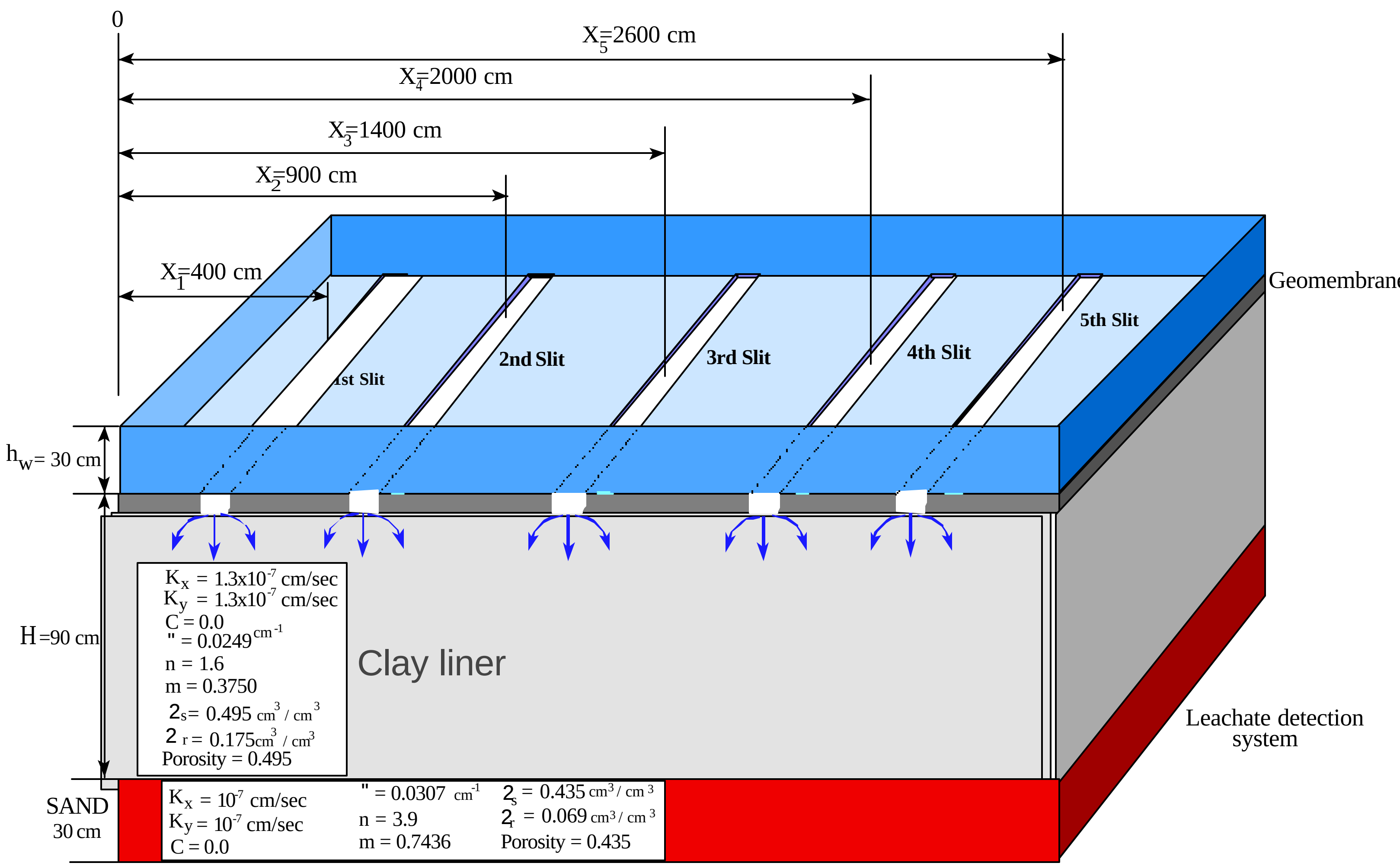
5) The hydraulic head at top of the clay barrier, just below a geomembrane slit is assumed to vary as described earlier (See Figure 3.15).

6) Locations of the multiple slits are chosen at 400 cm, 900 cm, 1400 cm, 2000 cm, and 2600 cm along x axis as shown in Figure 3.18.

Physical properties of the different types of soil materials are given in Table 3.1 and Table 3.2 in this chapter.

Table 3.2 Physical properties of the soil materials (Istok, 1989).

Material	Hydraulic Conductivity, K_s (m/sec)	Specific Storage, S_s (m^{-1})	Porosity, n	Bulk Density, ρ_b (kg/m^3)
Sand	$10^{-2} – 10^{-6}$	0.1 – 0.4	0.25 – 0.55	1300-1900
Silt	$10^{-3} – 10^{-7}$	0.2 – 0.4	0.35 – 0.60	1200-1800
Clay	$10^{-7} – 10^{-10}$	0.05 – 0.2	0.35 – 0.55	1000-1600



* Slit sizes are exaggerated.

Figure 3.18 Locations of the slits and properties of the soil

Equipotential lines of the two-dimensional case are given in Figure 3.19. There is horizontal flow between the geomembrane and the underlying clay barrier. Phreatic surface separates the soil which is not saturated from the soil where no flow occurs. The flow in the soil is assumed to be vertical in the plane and R is the width of the wetting area.

A two-dimensional soil profile was analyzed using the method described in Chapter 3. The finite element mesh employed is shown in Figure 3.20. To improve results around the second slit (distance from the slit center, X_1 = 900 cm), the mesh was refined. This mesh was used for all the examples in the study.

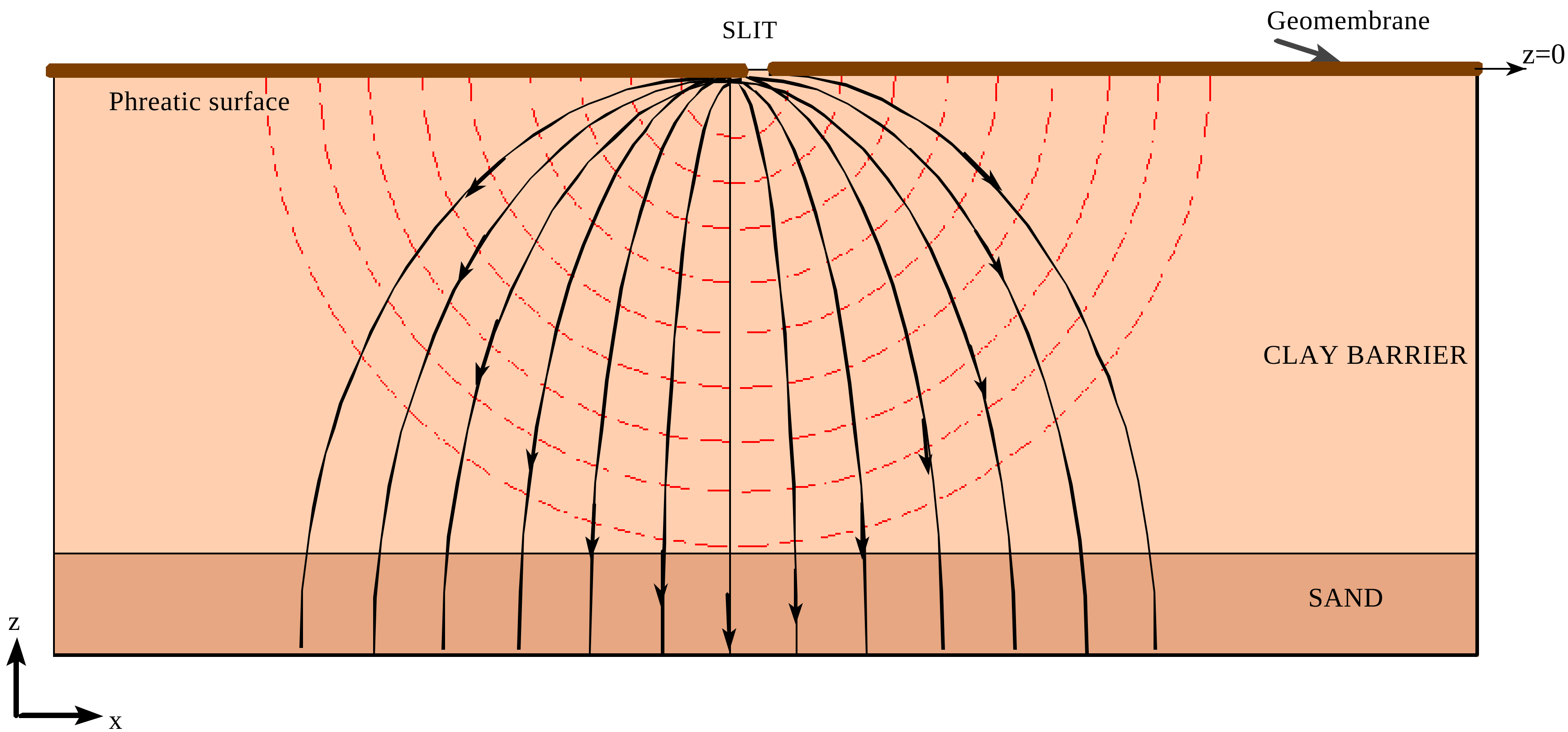


Figure 3.19 Equipotential flow lines of the leachate through a composite barrier.

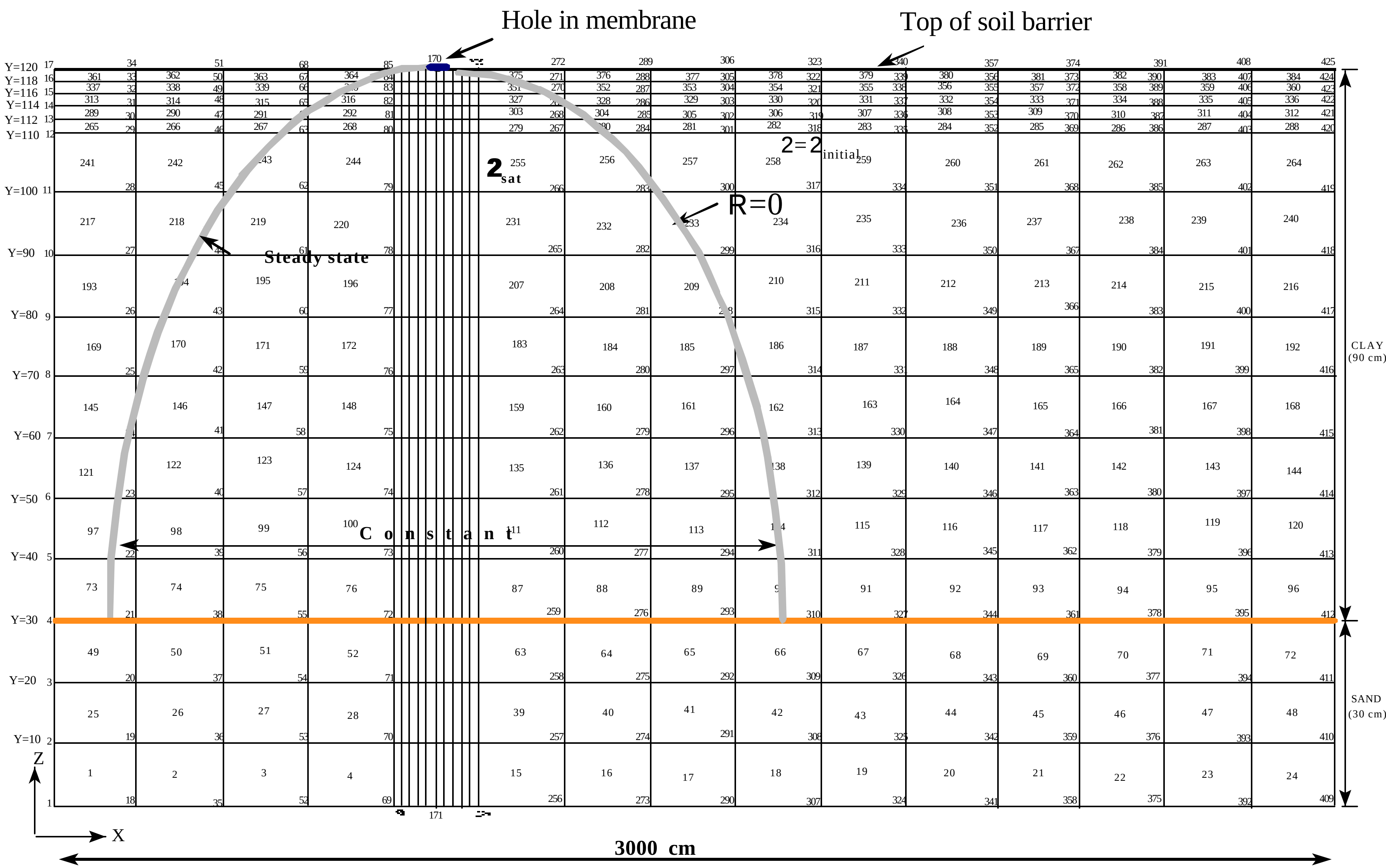


Figure 3.20 Mesh of the composite liner profile (Geomembrane, underlain by 90 cm of soil and 30 cm of sand, liquid detection system) and refined geomembrane hole area, the distance from z axis, x = 900 cm.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Overview

The computer code as described in Chapter 3 was used to calculate hydraulic heads, breakthrough times for the wetting front, and seepage fluxes for two cases in order to evaluate the accuracy of the model. The outcomes of the COMPBAR program simulations are presented in Section 4.2. The results of this study are compared with SOILINER and SEEP models in Sections 4.3 and 4.4, respectively. Finally, a stability and parametric analysis of the COMPBAR computer code are discussed in Sections 4.5 and 4.6, respectively.

Examples

In this section, two examples are presented. The complete input and output data files of Example 1 are given in the Appendix A.

Example 1

In this example, leakage through a composite barrier due to five slits in the geomembrane is considered. The geomembrane is assumed to be in good contact with the underlying clay. The selected values of pressure head (ψ), volumetric moisture content (θ), hydraulic conductivity (K), and flow rate (q) beneath the geomembrane slit for Example 1 are given in Tables 4.1 through 4.4 and Figures 4.1 through 4.4. These values are plotted for the clay layer of the composite barrier.

Table 4.1 Selected values of the pressure head (R) with depth below the slit of Example 1 (the distance from z axis, x=900 cm).

El.No.	x (cm)	z(cm)	4.0	4.8	9.0	13.4	16.0	18.0	20.6	26.0 (years)
170	900	90	-81.28	-4.57	-2.94	0.06	0.17	0.30	0.42	0.78
169	900	80	-75.03	-27.21	-18.96	-8.37	-8.03	-7.60	-7.21	-6.10
168	900	70	-75.02	-61.61	-39.10	-16.55	-15.85	-14.97	-14.15	-11.97
167	900	60	-75.00	-75.11	-74.91	-40.35	-36.70	-32.48	-29.24	-22.49
166	900	50	-74.16	-12.44	-8.97	-3.40	-3.20	-2.96	-2.74	-2.11
165	900	40	-36.39	0.35	1.09	2.83	2.92	3.03	3.13	3.41
164	900	30	-1.94	4.16	4.74	6.06	6.13	6.21	6.29	6.51
163	900	20	4.64	8.94	9.33	10.24	10.29	10.34	10.39	10.54
162	900	10	14.03	16.10	16.29	16.75	16.77	16.80	16.82	16.90
161	900	8	16.44	18.09	18.24	18.61	18.63	18.65	18.67	18.73
160	900	6	19.15	20.38	20.50	20.77	20.79	20.80	20.82	20.86
159	900	4	22.23	23.05	23.13	23.31	23.32	23.33	23.34	23.37
158	900	2	25.80	26.21	26.25	26.34	26.34	26.35	26.35	26.37
157	900	0	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00

Table 4.2 Selected values of the volumetric moisture content (2) with depth below the slit of Example 1 (the distance from z axis, x=900 cm).

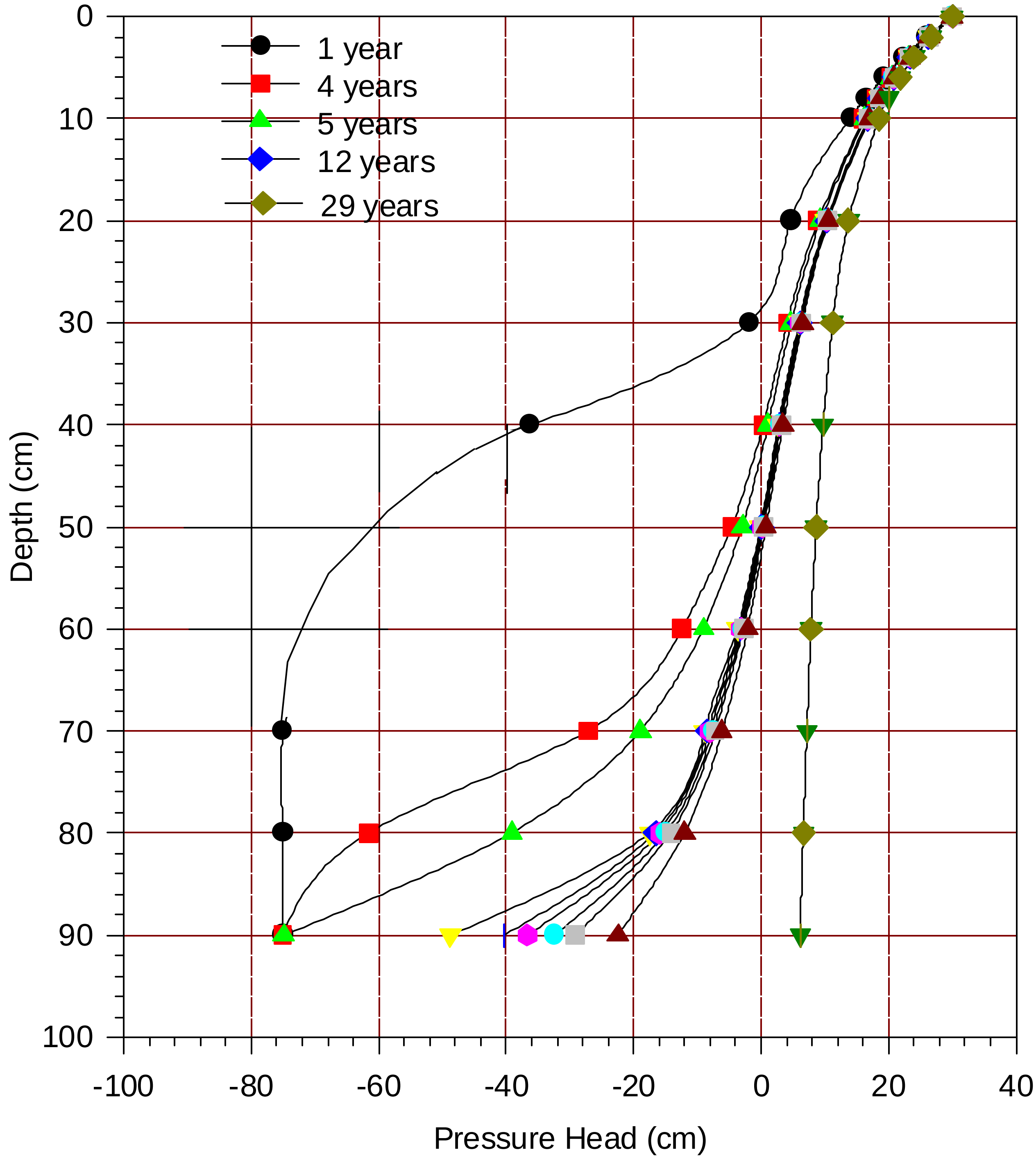
El.No.	z(cm)	z(cm)	1.3	4.0	4.8	9.0	12.4	16.0	18.2	26.0 (years)
369	890	85	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
345	890	75	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
321	890	65	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
297	890	55	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
273	890	45	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
249	890	35	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
225	890	25	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
201	890	15	0.4574	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
177	890	9	0.3875	0.4935	0.4945	0.4950	0.4950	0.4950	0.4950	0.4950
153	890	7	0.3677	0.4842	0.4887	0.4939	0.4941	0.4943	0.4944	0.4950
129	890	5	0.3711	0.4600	0.4736	0.4886	0.4894	0.4901	0.4905	0.4950
105	890	3	0.3705	0.4103	0.4400	0.4763	0.4782	0.4801	0.4812	0.4950
81	890	1	0.3706	0.3770	0.3921	0.4346	0.4462	0.4541	0.4584	0.4950

Table 4.3. Selected values of hydraulic conductivity (K) with depth below the slit of Example 1 (the distance from z axis, x=900 cm).

El.No.	x (cm)	z(cm)	1.3	4.0	6.1	11.6	14.0	18.4	23.5	25.2 (years)
369	890	85	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
345	890	75	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
321	890	65	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
297	890	55	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
273	890	45	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
249	890	35	0.0022	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
225	890	25	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
201	890	15	0.0012	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
177	890	9	0.0015	0.0112	0.0084	0.0112	0.0112	0.0112	0.0112	0.0112
153	890	7	0.0002	0.0040	0.0050	0.0078	0.0079	0.0081	0.0092	0.0112
129	890	5	0.0021	0.0016	0.0026	0.0051	0.0052	0.0053	0.0062	0.0112
105	890	3	0.0001	0.0004	0.0009	0.0029	0.0030	0.0032	0.0042	0.0112
81	890	1	0.0001	0.0001	0.0002	0.0009	0.0010	0.0012	0.0022	0.0112

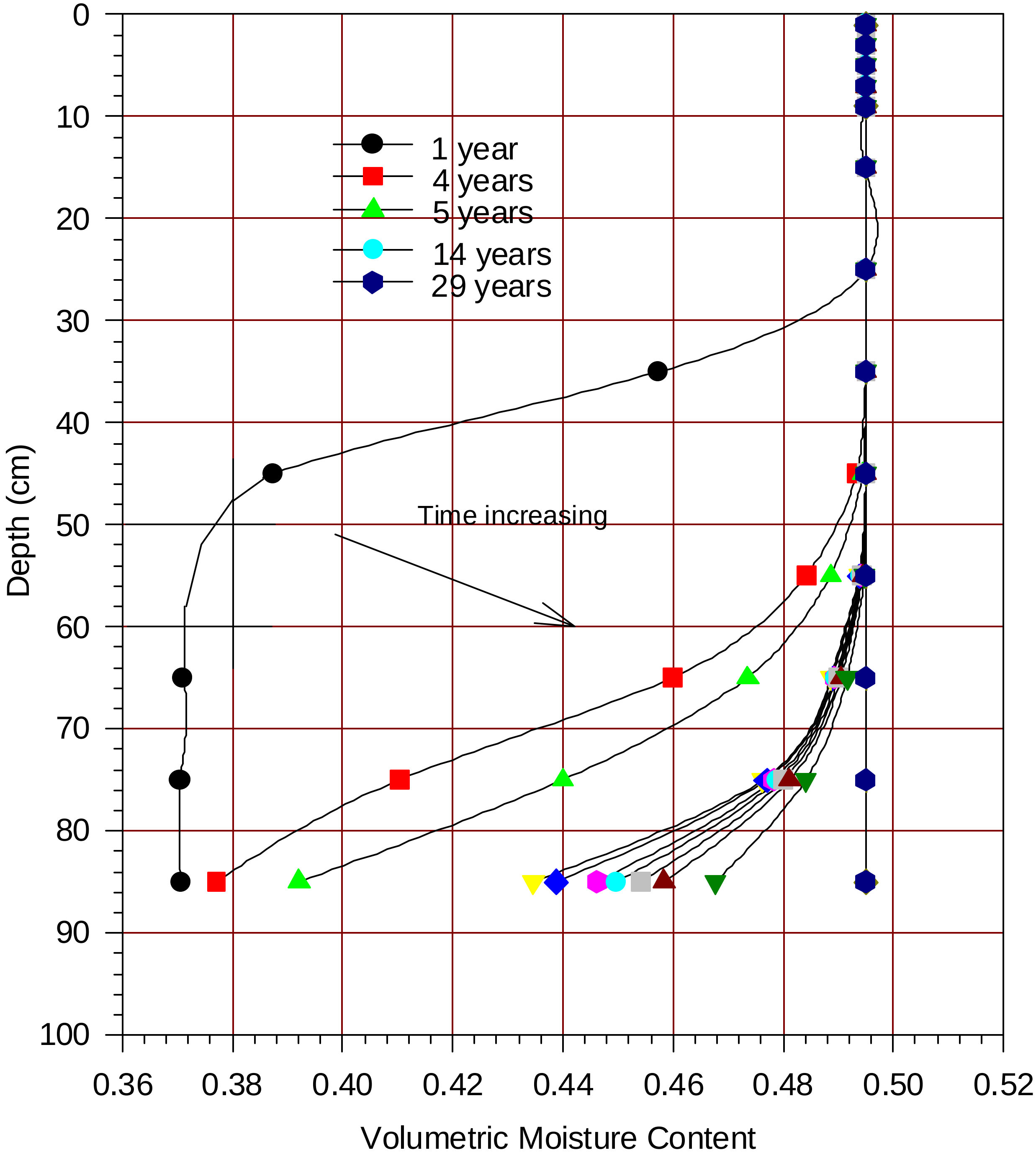
Table 4.4. Selected values of flow rate (q) with depth below the slit of Example 1 (the distance from z axis, x=900 cm).

El.No.	x (cm)	z(cm)	1.3	4.0	0.8	0.2	1.6	4.0	0.6	26.0 (years)
369	890	85	2.1547	2.1771	2.1844	2.1844	2.1846	.1850	2.1855	2.1863
345	890	75	1.9247	1.9921	1.9986	2.0140	2.0147	2.0158	2.0174	2.0196
321	890	65	1.7101	1.8225	1.8333	1.8590	1.8601	1.8619	1.8646	1.8683
297	890	55	1.5085	1.6658	1.6809	1.7169	1.7185	1.7210	1.7247	1.7300
273	890	45	1.3178	1.5201	1.5395	1.5858	1.5878	1.5910	1.5958	1.6026
249	890	35	0.1635	0.2310	0.2374	0.2527	0.2534	0.2544	0.2560	0.2583
225	890	25	0.0082	0.1248	0.1355	0.1607	0.1618	0.1635	0.1662	0.1699
201	890	15	-0.0673	0.0373	0.0527	0.0868	0.0883	0.0907	0.0943	0.0996
177	890	9	-0.0227	-0.0386	-0.0235	0.0234	0.0253	0.0283	0.0330	0.0396
153	890	7	-0.0152	-0.0735	-0.0653	-0.0319	-0.0303	-0.0278	-0.0235	-0.0168
129	890	5	-0.0164	-0.0691	-0.0762	-0.0646	-0.0635	-0.0618	-0.0587	-0.0534
105	890	3	-0.0162	-0.0344	-0.0543	-0.0.767	-0.0.767	-0.0765	-0.0758	-0.0736
81	890	1	-0.0162	-0.0185	-0.0248	-0.0534	-0.0.576	-0.0617	-0.0681	-0.0739



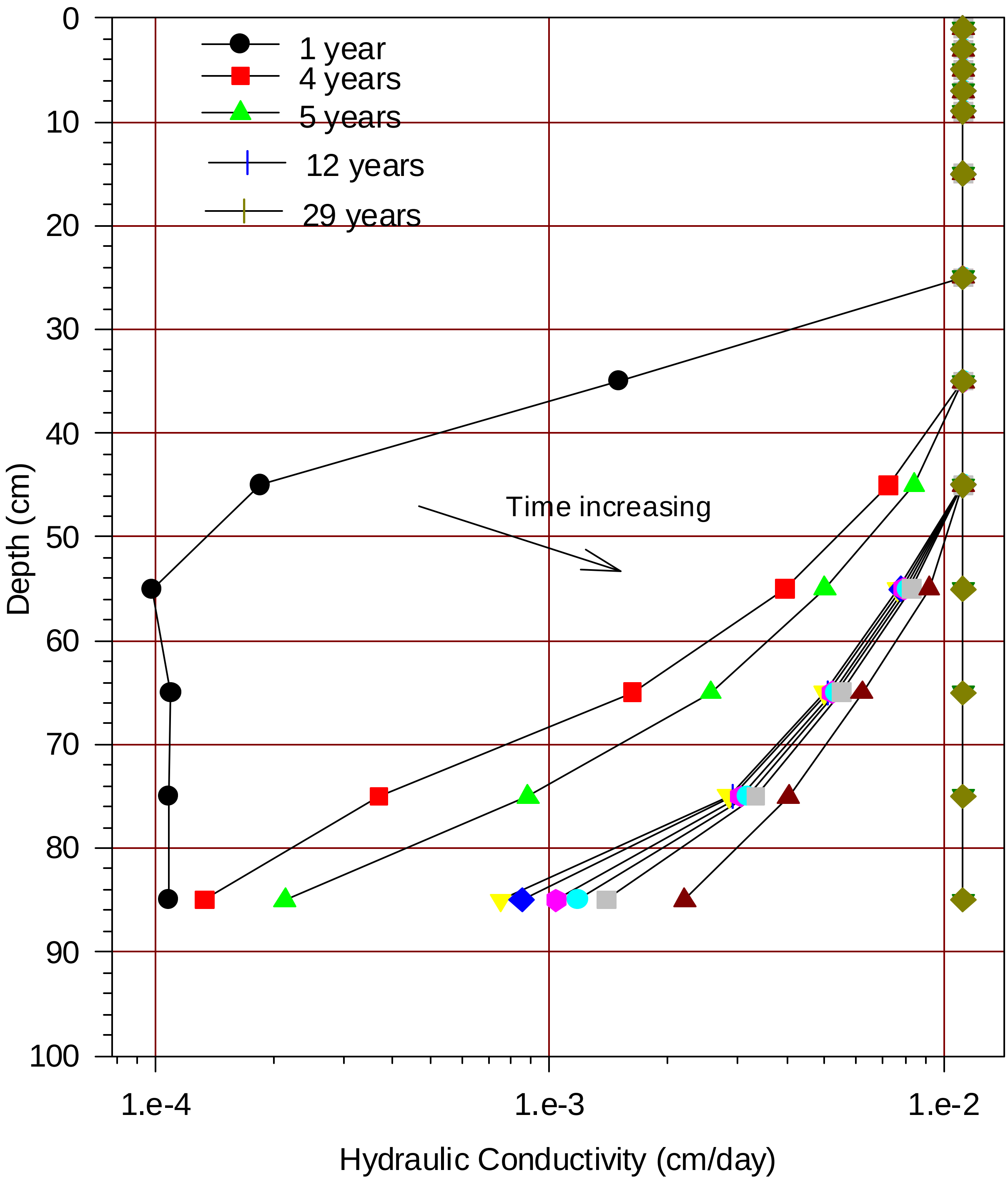
NOTE: The distance from z axis below the slit at x=900 cm.

Figure 4.1 Variation of pressure head with depth below the slit



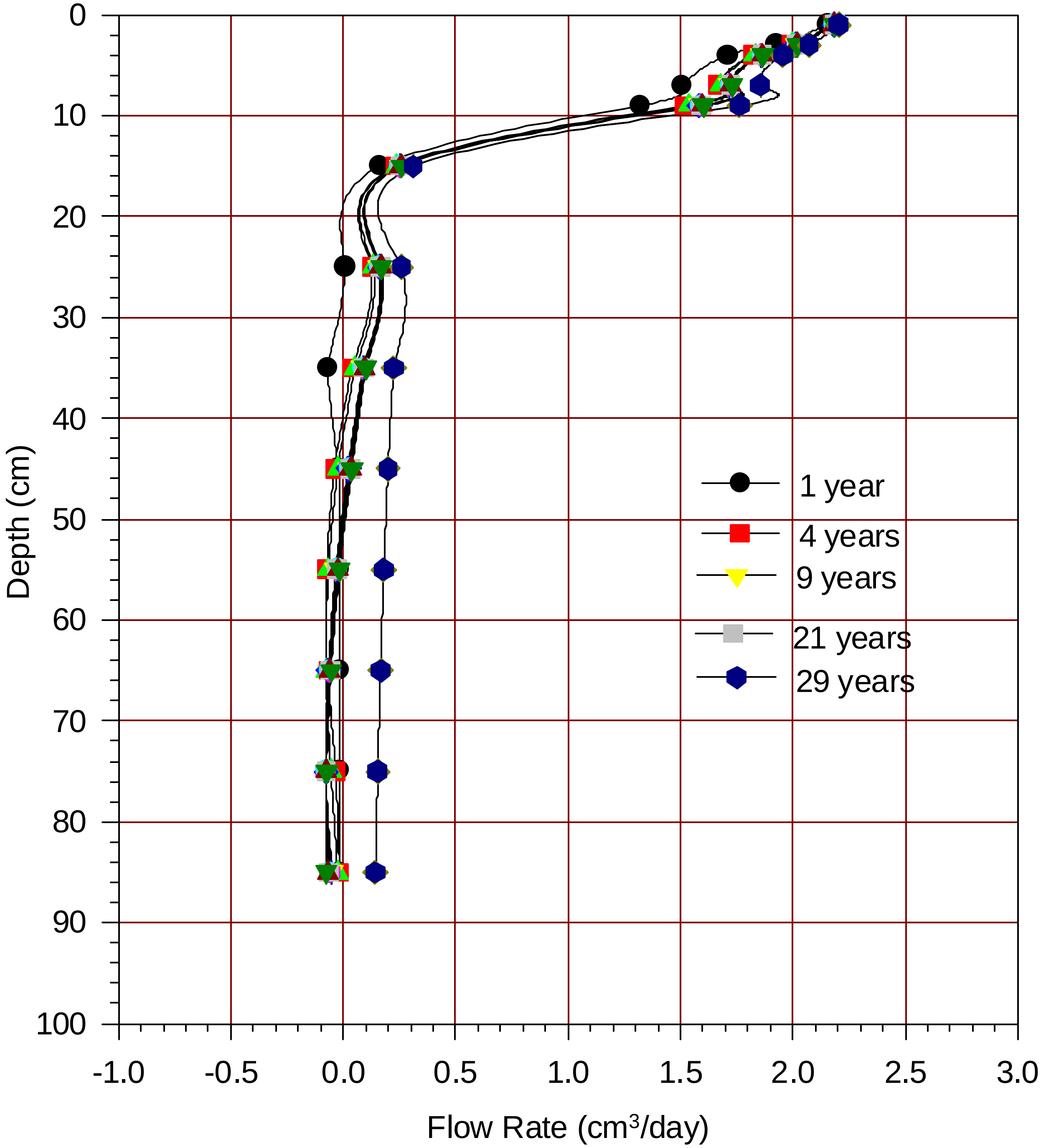
NOTE: The distance from z axis below the slit at x=900 cm.

Figure 4.2 Variation of the volumetric moisture content with depth below the slit



NOTE: The distance from z axis below the slit at x = 900cm.

Figure 4.3 Variation of hydraulic conductivity with depth below the slit of Example 1



NOTE: The distance from z axis below the slit at X=900 cm.

Figure 4.4 Variation of the flow rate with depth below the slit of Example 1

The values of pressure head (ψ), which are obtained at the nodes of the mesh, are presented in Table 4.1. These pressure heads are given at constant values of X (slit location of the geomembrane) and Y value is varied from 0 cm at the geomembrane to 120 cm at the base of the sand layer (leachate detection layer). In Figure 4.1, the pressure head (ψ) is maximum (positive) at the top of the clay layer and decreases in a nonlinear fashion to its minimum value, which is at the bottom of the clay layer (90 cm). It can be seen from Figure 4.1 that pressure head (ψ) values were initially –75 cm at the base of the clay layer and start shifting gradually to the right to approach zero and eventually to become positive after 29 years. The shift to the right indicates that the soil is moving from an unsaturated condition to a saturated condition. For example, at a level of 50 cm and after 1.0 year, pressure head (ψ) is –82 cm, and for the same depth after 29 years it is +8 cm, which means the soil profile was saturated at this depth. It can also be explained that after 1.0 year the soil is saturated on top of the clay barrier to depth of 30 cm, and beyond this depth the soil is still unsaturated. After 9 years the soil is saturated up to 50 cm and unsaturated below this depth. It is assumed that at the boundary (top of clay barrier) pressure head (ψ) stays constant with time, but moving away from the boundary, pressure head lines of Figure 4.1 start separating. After 29 years the entire soil profile was saturated and pressure head (ψ) values were all greater than zero. There are numerous different time intervals with various pressure head (ψ) values between the interval of 1 year and 29 years. As time increases the depth at which pressure head (ψ) values is greater than zero increases, which indicates that soil saturation increases with depth.

The values of volumetric moisture content (θ) at the center of each element are given in Table 4.2 and in Figure 4.2. Since volumetric moisture content (θ) is approximated from pressure head (ψ), which was obtained from the COMPBAR analysis, volumetric moisture content (θ) trends are similar to the pressure head (ψ) trends in the soil profile, i.e. maximum values occur at the top of the clay barrier and then decrease in a nonlinear fashion to minimum values at the base of the clay barrier. It can be seen from Figure 4.2 that up to a depth of 25 cm the soil has volumetric moisture content (θ) level equal to the saturated volumetric moisture content level at 29 years, which indicates fully saturated conditions. The volumetric moisture content (θ) values are lower than the saturated θ values below a depth of 30 cm. Therefore, with increasing time, the unsaturated clay approaches saturated conditions. For example, after 4 years the soil is fully saturated up to a depth of 45 cm and is unsaturated below that point. After 29 years, volumetric moisture content (θ) is 0.495 throughout the soil profile, which is equal to porosity, and indicates that all the pores in the soil are saturated.

The values of hydraulic conductivity (K) approximated at the center of the each element are given in Table 4.3. A selected part of this output at geomembrane slit located at x=900 cm (see Figure 3.18) is plotted in Figure 4.3. Hydraulic conductivity (K) values are maximum, about 1.1 x 10^{-3} cm/day, at the top, and minimum of about 2.0x10^{-4} cm/day at a depth of 85 cm into the clay barrier. In general, saturated hydraulic conductivity (K) values are higher than unsaturated values due to the presence of air bubbles within the soil matrix which impede flow, thus yielding lower K values. Initially, the clay layer is not saturated, and as time progresses the soil's volumetric water content increases which results in larger hydraulic conductivity values. For example, after one year, the depth of 35 cm hydraulic conductivity is 1.5 x 10^{-3} cm/day. After one year, and at a depth of 45 cm, hydraulic conductivity the value is 3.0 x 10^{-4} cm/day and for the same depth after four years hydraulic conductivity is 7.0 x 10^{-3} cm/day. After four years, hydraulic conductivity value at 45 cm is higher because of a higher saturation level then after twenty six years hydraulic conductivity is 1.1x10^{-2} cm/day, which is equal to the saturated hydraulic conductivity.

Table 4.4 shows the approximated values of flow rate in the elements. These values are also plotted in Figure 4.4. Initially, flow rate has a peak value of 2.2 cm^3/day at the top of the soil profile because the soil is saturated. Then, flow rate drops drastically to -0.3 cm^3/day at a depth of 15 cm under steady state conditions with no significant change below 15 cm. After one year, since the values of hydraulic conductivity are low, the leachate has reached the leachate detection system. Beyond the depth of 20 cm, flow rate increases with increasing time. For example, at the depth of 25 cm after fourteen years, flow rate was about -0.2 cm^3/day whereas after twenty six years it was about 0.3 cm^3/day.

The wetting front movement of this example is plotted in Figure 4.5. The wetting front was produced by plotting contour lines of equal pressure head (ψ) in the soil layer. The graph

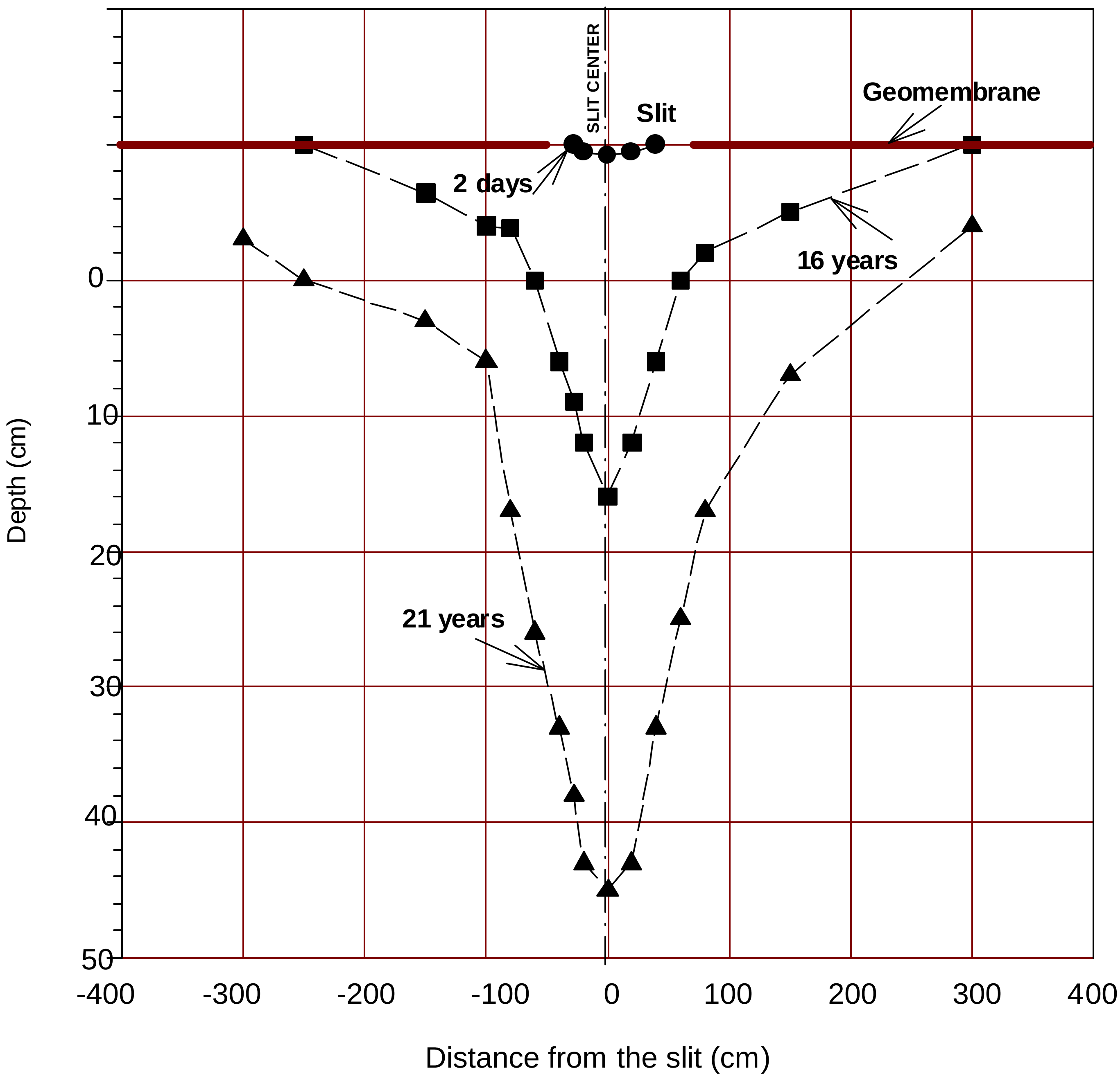


Figure 4.5 Wetting front movement with different time intervals

indicates the location of the leachate moving down through the soil profile. It can be seen from Figure 4.5 that after two days the wetting front remains just beneath the geomembrane. After sixteen years the wetting front directly under a slit reached a depth of 18 cm but decreases laterally away form the hole. After twenty one years the wetting front reached a depth of 45 cm and also decreased away from the centerline of the hole.

Example 2

In the second example, only one slit in the geomembrane was considered, and the geomembrane had poor contact with the underlying clay barrier. The selected output values and graphs for Example 2 are presented in Tables 4.5 through 4.8 and Figures 4.6 through 4.9. At the location of the slit, similar to Example 1, x values are kept constant and z values are changed from 0 to 120 cm below the slit.

In Table 4.5, values of pressure heads (ψ) are listed at different time intervals at nodes of the mesh. Part of this output is plotted in Figure 4.6. This figure shows that the pressure at the top of the soil barrier is at a maximum, and it decreases nonlinearly to a minimum value at the bottom of the clay barrier. After 1 year at a depth of 50 cm, pressure head was –85 cm. After 25 years pressure head at the same depth was –0.2 cm. After 1 year, the soil is saturated between 0 cm to 28 cm. Beyond this depth the soil remains unsaturated. After 4 years the soil is saturated to a depth of 44 cm and unsaturated below this depth.

The values of volumetric moisture content (θ) at the center of each element are presented in Table 4.6 and in Figure 4.7. Figure 4.7 shows that, after one year, the soil profile was saturated (i.e., $\theta = 0.495$) up to a depth of 28 cm, then volumetric moisture content (θ) value sharply decreases to about 0.37 at depths between 55 cm to 85 cm. As time increases, the saturation front approaches full saturation. For example, after four years, θ at a depth of 65 cm increases to 0.472. Then, after twenty nine years volumetric moisture content value increases to 0.495 indicating full saturation.

Table 4.5 Selected values of pressure head (R) with depth below the slit of Example 2 (the distance from z axis, z=900 cm).

El.No.	x (cm)	z (cm)	4.0	4.8	9.0	13.4	16.0	8.0	20.6	26.0 (years)
170	900	90	-75.00	-75.09	-69.88	-56.25	-50.38	-47.41	-34.67	-33.33
169	900	80	-75.05	-51.89	-15.65	-14.56	-14.78	-14.83	-13.78	-13.54
168	900	70	-74.91	-20.52	-7.56	-.6.61	-6.72	-6.73	-6.21	-6.10
167	900	60	-74.51	-8.72	-2.52	-1.86	-1.91	-1.92	-1.61	-1.55
166	900	50	-80.83	-2.14	-0.89	1.26	1.23	1.24	1.37	1.40
165	900	40	-26.76	1.84	3.90	4.23	4.22	4.22	4.33	4.36
164	900	30	-1.45	5.79	7.33	7.61	7.60	7.60	7.69	7.71
163	900	20	5.60	10.56	11.59	11.79	11.78	11.78	11.84	11.85
162	900	10	14.92	17.36	17.87	17.97	17.97	17.97	18.00	18.01
161	900	8	17.26	19.21	19.62	19.70	19.70	19.70	19.72	19.73
160	900	6	19.85	21.31	21.62	21.68	21.68	21.70	21.70	21.70
159	900	4	22.77	23.74	23.94	23.99	23.99	23.99	24.00	24.00
158	900	2	26.11	26.60	26.70	26.72	26.72	26.72	26.73	26.72
157	900	0	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00

Table 4.6 Selected values of volumetric moisture content (2) with depth below the slit of Example 2 (the distance from z axis, z=900 cm).

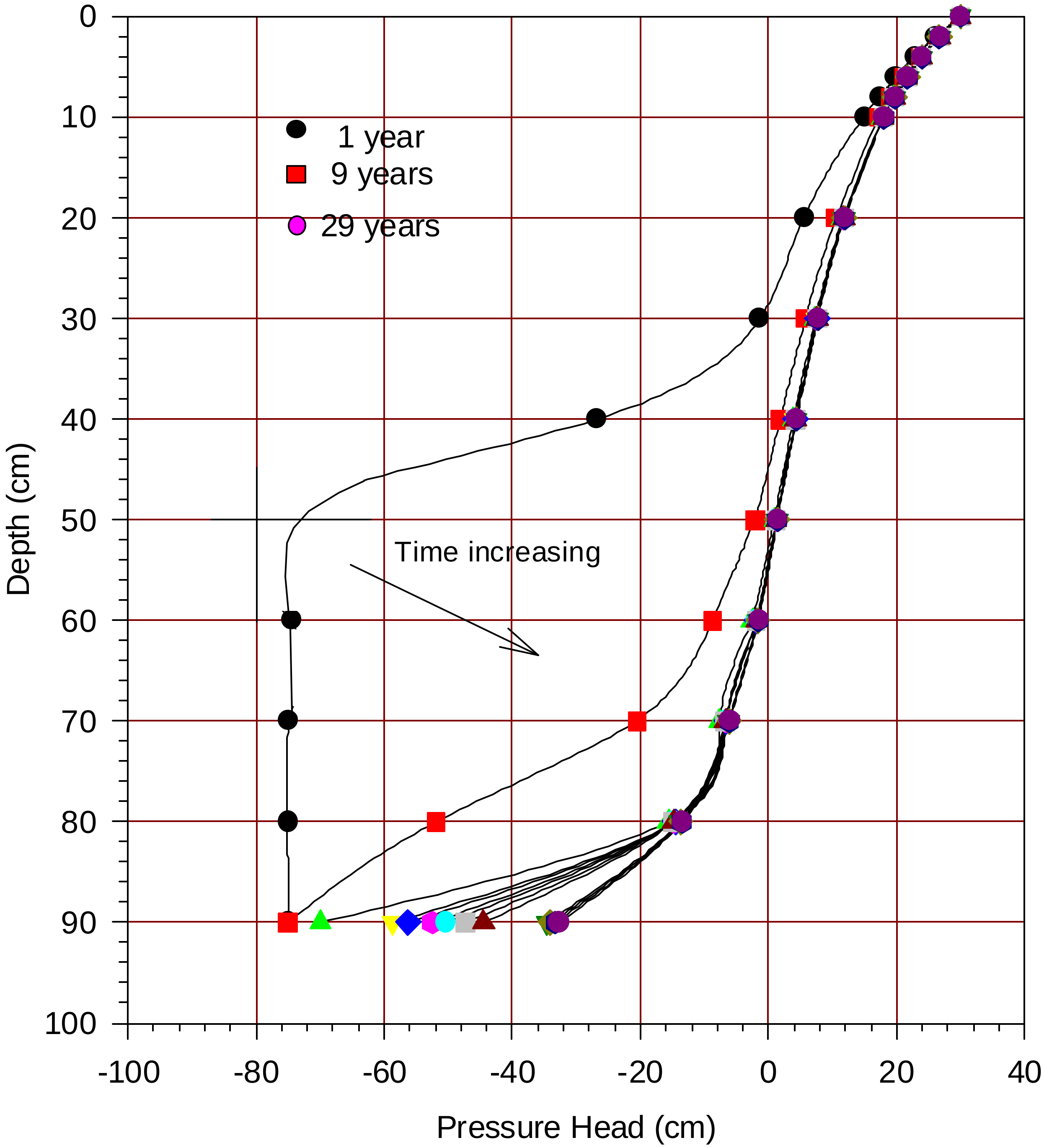
El.No.	x (cm)	z (cm)	1.3	4.0	6.2	9.5	11.2	15.3	17.3	26.7 (years)
369	890	85	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
345	890	75	0.4950	0.4950	0.9550	0.4950	0.4950	0.4950	0.4950	0.4950
321	890	65	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
297	890	55	0.4950	0.4950	.0950	0.4950	0.4950	0.4950	0.4950	0.4950
273	890	45	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
249	890	35	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
225	890	25	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
201	890	15	0.4692	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
177	890	9	0.3948	0.4949	0.4950	0.4950	0.4950	0.4950	0.4950	0.4950
153	890	7	0.3675	0.4894	0.4946	0.4949	0.4949	0.4949	0.4949	0.4950
129	890	5	0.3710	0.4720	0.4903	0.4915	0.4915	0.4914	0.4914	0.4921
105	890	3	0.3706	0.4254	0.4792	0.4815	0.4814	0.4811	0.4811	0.4824
81	890	1	0.3706	0.3830	0.4165	0.4280	0.4308	0.4372	0.4401	0.4549

Table 4.7 Selected values of hydraulic conductivity (K) with depth below the slit of Example 2 (the distance from z axis, z=900 cm).

El.No.	x (cm)	z (cm)	1.3	4.0	6.1	11.6	14.0	18.4	23.5	25.2 (years)
369	890	85	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
345	890	75	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
321	890	65	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
297	890	55	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
273	890	45	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
249	890	35	0.0022	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
225	890	25	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
201	890	15	0.0022	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
177	890	9	0.0098	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
153	890	7	0.0054	0.0088	0.0097	0.0097	0.0096	0.0096	0.0099	0.0102
129	890	5	0.0025	0.0056	0.0061	0.0061	0.0060	0.0060	0.0062	0.0063
105	890	3	0.0006	0.0032	0.0035	0.0035	0.0035	0.0035	0.0036	0.0137
81	890	1	0.0002	0.0004	0.0006	0.0007	0.0008	0.0008	0.0012	0.0014

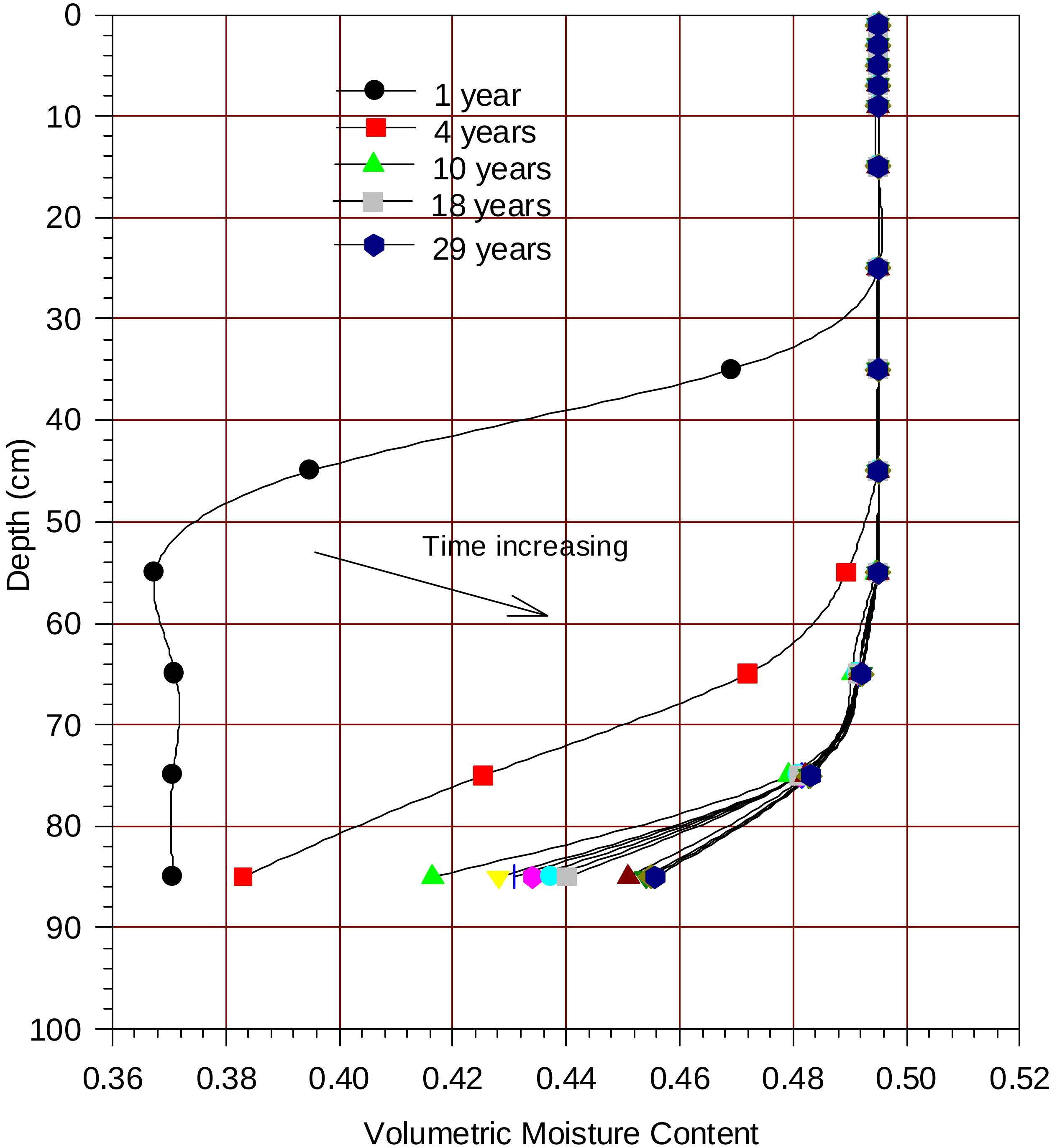
Table 4.8 Selected values of flow rate (q) with depth below the slit of Example 2 (the distance from z axis, x=900 cm).

El.No.	x (cm)	z (cm)	1.3	4.0	4.8	10.2	11.6	14.0	23.5	25.2 (years)
369	890	85	2.3041	2.3309	2.3365	2.3377	2.3377	2.3377	2.3380	2.3381
345	890	75	2.0774	2.1577	2.1745	2.1783	2.1782	2.1783	2.1790	2.1794
321	890	65	1.8639	1.9978	2.0259	2.0321	2.0320	2.0321	1.0332	1.0339
297	890	55	1.6616	1.8491	1.8884	1.8971	1.8969	1.8971	1.8987	1.8996
273	890	45	1.4686	1.7100	1.7605	1.7716	1.7714	1.7716	1.7737	1.7749
249	890	35	0.1898	0.2709	0.2877	0.1913	0.2912	0.2913	0.2020	0.2924
225	890	25	0.0274	0.1659	0.1937	0.1996	0.1994	0.1996	0.2007	0.2014
201	890	15	-0.0746	0.0770	0.1157	0.1234	0.1232	0.1234	0.1249	0.1258
177	890	9	-0.0261	-0.0084	-0.0471	0.0559	0.0557	0.0559	0.0578	0.0590
153	890	7	-0.0152	-0.0615	-0.0190	-0.0099	-0.0102	-0.0074	-0.0067	-0.0056
129	890	5	-0.0163	-0.0761	-0.0595	-0.0539	-0.0541	-0.0541	-0.0522	-0.0513
105	890	3	-0.0162	-0.0460	-0.0765	-0.0757	-0.0.758	-0.0758	-0.0752	-0.0749
81	890	1	-0.0162	-0.0215	-0.0381	-0.0480	-0.0510	-0.0558	-0.0626	-0.064



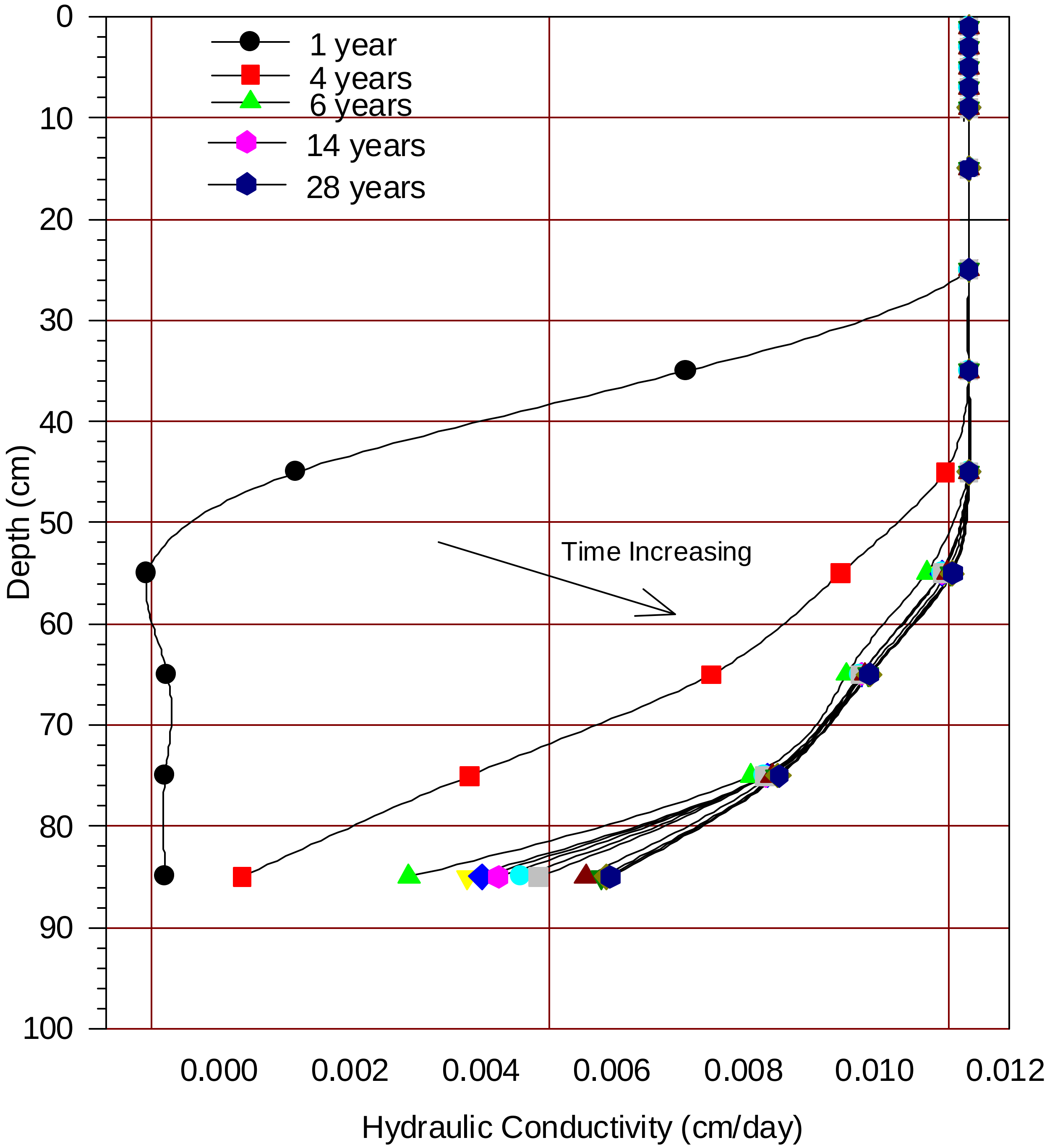
NOTE: The distance from z axis below the slit x=900 cm.

Figure 4.6 Variation of pressure head with depth below the slit of Example 2



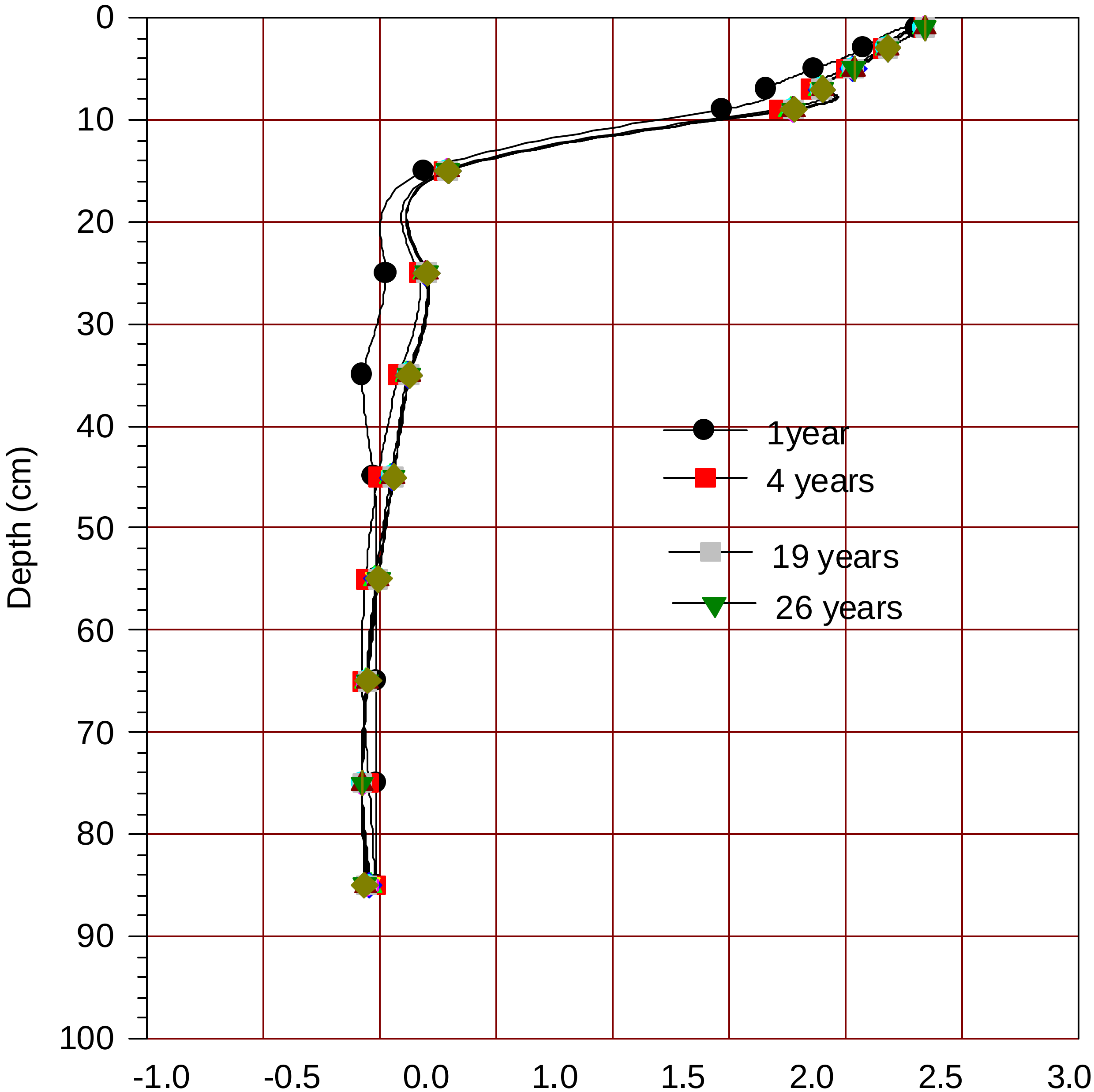
NOTE: The distance from z axis below the slit at x=900 cm.

Figure 4.7 Variation of the volumetric moisture content with depth below the slit of Example 2



NOTE: The distance from z axis below the slit at x=900 cm.

Figure 4.8 Variation of hydraulic conductivity with depth below the slit of Example 2



Flow Rate (cm^3/day)

NOTE: The distance from z axis below the slit at x=900 cm.

Figure 4.9 Variation of flow rate (q) with depth below the slit of Example 2

Hydraulic conductivity (K) values for Example 2 are given in Table 4.7 and Figure 4.8. After 1 year at the depth of 35 cm, hydraulic conductivity is about 2.0 x 10^{-3} cm/day. For the same depth after 4 years, hydraulic conductivity is 1.1 x 10^{-2} cm/day, and after 25 years it is the same.

The selected values of flow rate (q) for Example 2 are given in Table 4.8. These values are plotted in Figure 4.9. Flow lines of the wetting front at 21,000 sec with 30 cm and 70 cm leachate heads are given in Figure 4.10 and 4.11, respectively. It can be seen from these figures that the flow lines are closer at the bottom than at the top locations. Both figures show that the higher the pressure head, the greater initial lateral spreading which is followed by a narrowing as it migrates downward.

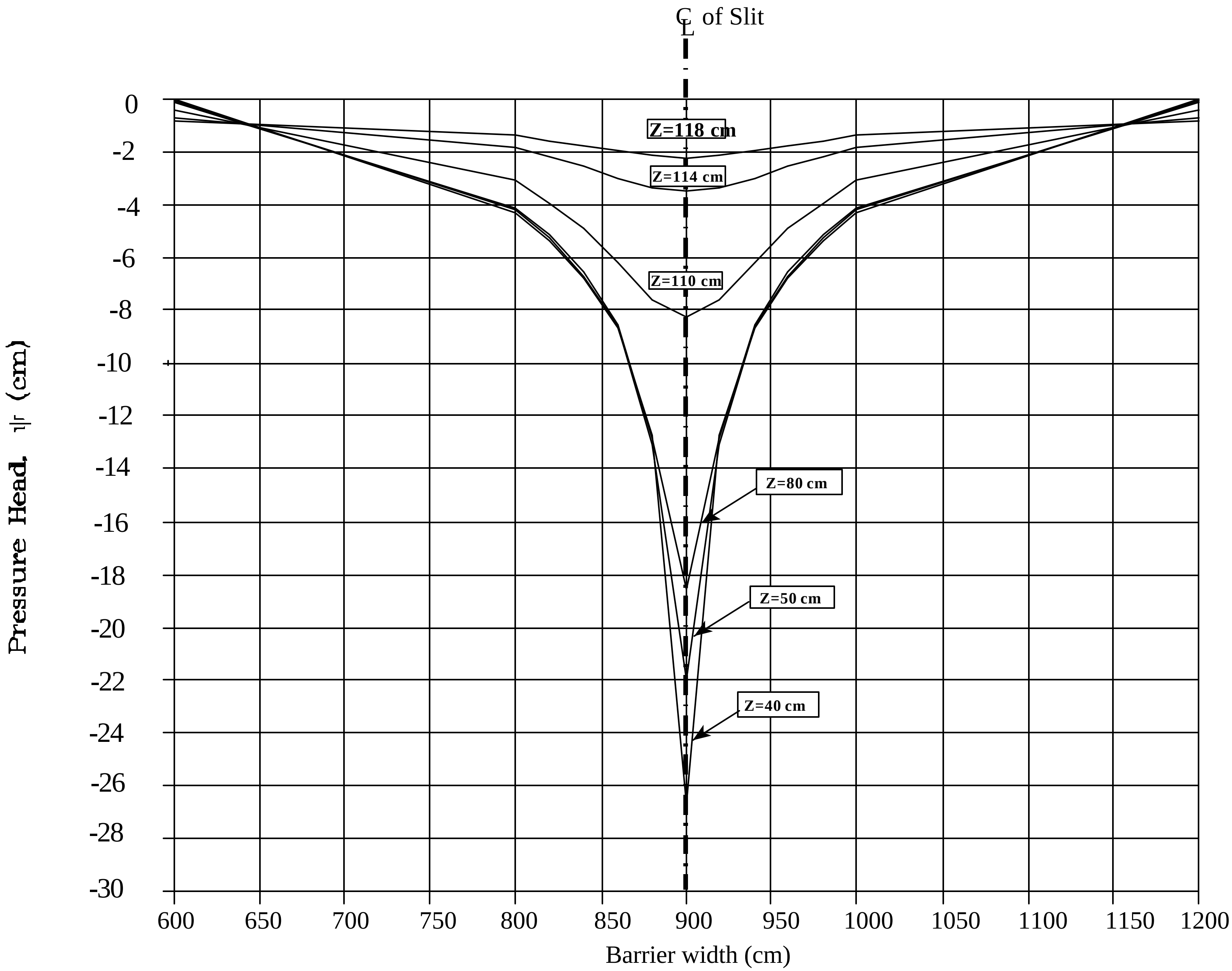


Figure 4.10 Flow lines of wetting front at 21,000 seconds (6 hours) (h_w = 30 cm)

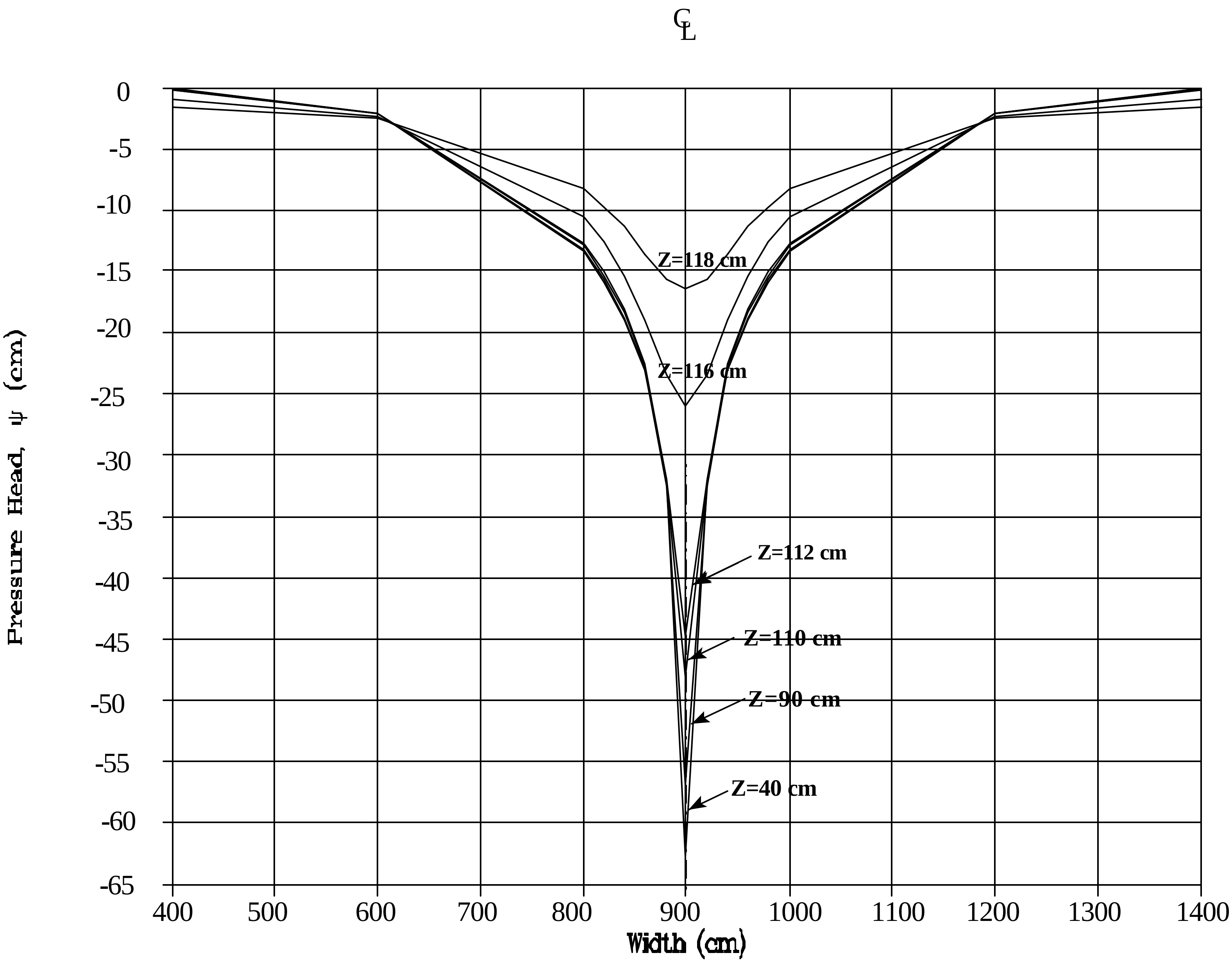


Figure 4.11 Flow lines of the wetting front at 21,000 sec. (6 hours) (h_w = 70 cm)

4.2 Results of COMPBAR

Two empirical equations for the determination of flow rate through the soil profile for good and poor contact conditions were developed by Giroud et al. (1989 a, b) and are presented in Equation 2.1 and 2.2 in Chapter 2. These equations were coupled with the finite element method to analyze the transient flow through an unsaturated compacted clay barrier. The technique can also be used to predict the time required for a wetting front to migrate completely through the clay barrier. This time will determine the effective life of a composite barrier. The ability to determine the life of a composite barrier system has significant implications for forecasting the time that a landfill can be used efficiently, and for preventing the contamination of the ground water table. To determine when the wetting front reaches the leachate detection layer, the nodes in the mesh which are located adjacent to the sand layer should be monitored closely. When the nodes have attained a pressure head (ψ) value of zero, this indicates that saturated conditions and the wetting front have reached the leachate detection layer. As a verification, the results obtained from this technique were compared with results reported by Giroud et. al. (1989). The calculation of overall flow rate (Q) values from the empirical equations, flow rate values through the soil profile calculated using COMPBAR, and the time required to reach the leachate detection system for both cases are presented in Table 4.9.

The time estimation using the COMPBAR program was based on the time required time for the wetting front (pressure head values equal to zero) to reach the leachate detection layer. The time calculation using Girouds' results were based on Equation 3.1. It can be seen from data in Table 4.9 that, in most cases, the time estimation for the wetting front to reach the sand layer is about the same.

4.3 Comparison with SOILINER Results for a Hypothetical Case

The example given in the SOILINER code manual (Goode et al 1986) was analyzed using the technique developed in this study. A sketch of the barrier for this example was presented in Chapter 3, Section 3.5.1. Pressure head (ψ) and volumetric moisture content (θ) values from

Table 4.9 Time calculated by using Darcy's law, Q=kiA and the finite element method (i.e., COMPBAR computer code).

GIROUD'S PROCEDURE

Example Name (COMPBAR)	***Bulk value of Flow rate (Q) obtained by using Q=kiA (m^3/sec)***	***Time obtained from the bulk values of Flow rate (Q) (years)***	***Time calculated by finite element method (years)***
Example 1	$3.75x10^{-08}$	22.80	22.80
Example 2	$4.11x10^{-08}$	21.00	25.00
Example 3	$2.05x10^{-07}$	4.20	4.10
Example 4	$8.05x10^{-08}$	10.60	10.80
Example 5	$4.41x10^{-07}$	19.00	20.50
Example 6	$2.39x10^{-07}$	3.60	4.00
Example 7	$6.23x10^{-06}$	0.14	0.17
Example 8	$4.73x10^{-09}$	181.00	167.00
Example 9	$7.51x10^{-09}$	114.00	109.00
Example 10	$4.21x10^{-08}$	20.00	19.50
Example 11	$4.78x10^{-08}$	17.80	18.60
Example 12	$8.82x10^{-08}$	9.70	10.50
Example 13	$6.82x10^{-06}$	0.13	0.14
Example 14	$1.61x10^{-08}$	53.00	48.00
Example 15	$4.78x10^{-08}$	17.90	18.60
Example 16	$1.24x10^{-06}$	0.69	0.80
Example 17	$2.67x10^{-06}$	0.32	0.40
Example 18	$9.46x10^{-10}$	905.00	850.00
Example 19	$1.50x10^{-11}$	5110.00	5700.00
Example 20	$1.19x10^{-10}$	7189.00	7100.00

SOILINER and COMPBAR results are plotted along the depth shown in Figures 4.12 and 4.13, respectively. It is observed from these figures that pressure head and volumetric moisture content values estimated from SOILINER computer programs are similar and have similar trends. As seen in Figure 4.13, both programs show that the leachate migrates vertically, then laterally, and vertically again. Also the pressure head behaves in a similar fashion, increasing as the depth increases, Figure 4.12.

It can be seen from the Table 4.9 that in most cases, the time estimated for the wetting front to reach the sand layer is almost the same. As a matter of fact that the times obtained from the COMPBAR program are more reliable because the flow rate changes with the depth as a function of time.

4.4 Comparison with SEEP Program

Another type of verification has been conducted assuming steady-state, saturated conditions. The initial degree of saturation of the soil profile was assumed as 100 %. The variations of total stress with depth for a saturated soil and leachate placed on top of the soil profile are shown graphically in Figure 4.14 (a) and (b). Figure 4.14 a shows 90 cm of saturated soil, with the groundwater table at the top. For this case, total stress is zero at the top of the saturated soil and increases linearly with depth. Figure 4.14 b shows 90 cm of an unsaturated soil liner with 30 cm of leachate placed on top of soil liner. The same mesh, as shown in Figure 3.20, was used. The initial pressure head (ψ) values were assumed as zero, which indicates saturation, and several cases were analyzed. A comparison of COMPBAR and SEEP computer results was made to show that both programs give similar linear results with a saturated soil profile. Only COMPBAR, however, can provide results for a geomembrane placed on top of a slit, under unsaturated soil conditions.

The first case assumed that there was no geomembrane between the leachate impoundment and the clay barrier. The results for this case are shown in Figure 4.15. The variation of pressure head with depth is linear to a depth of 30 cm at the top of the clay barrier, and continues linearly to a depth of 90 cm. The same result was also obtained using the SEEP program.

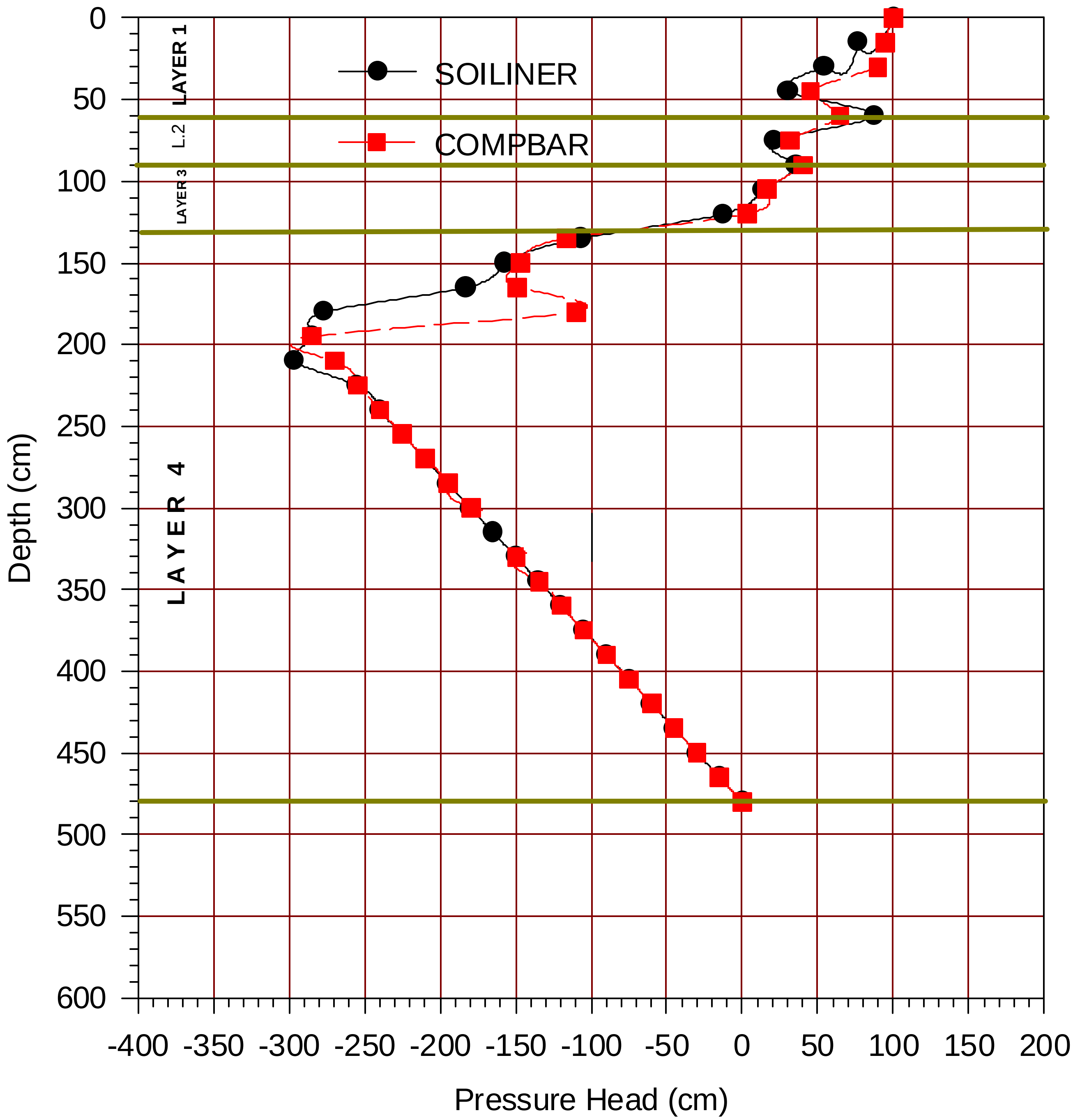


Figure 4.12 Variation of presssure head for Example 1 by using both SOILINER and COMPBAR programs (after 1 year)

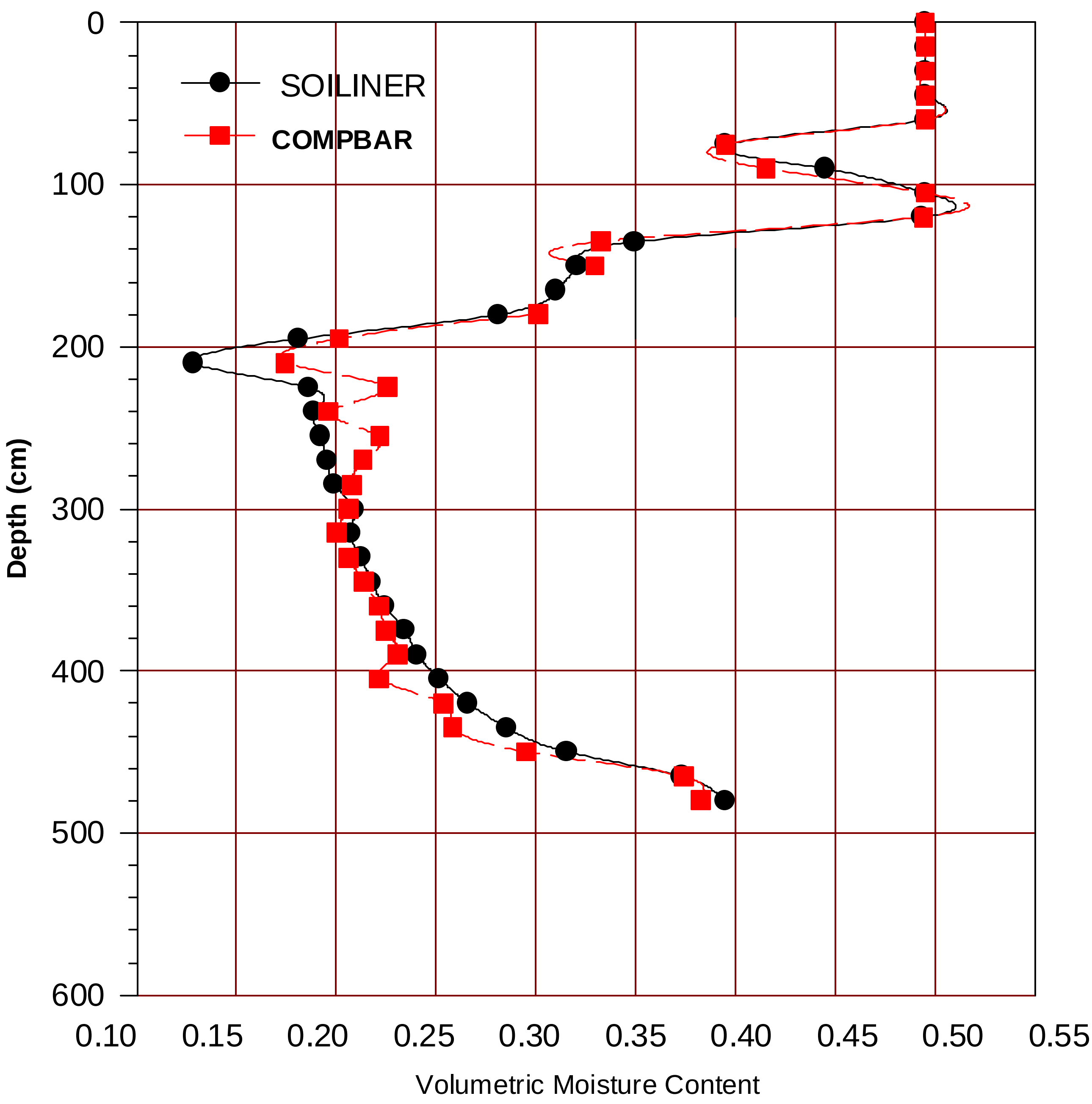
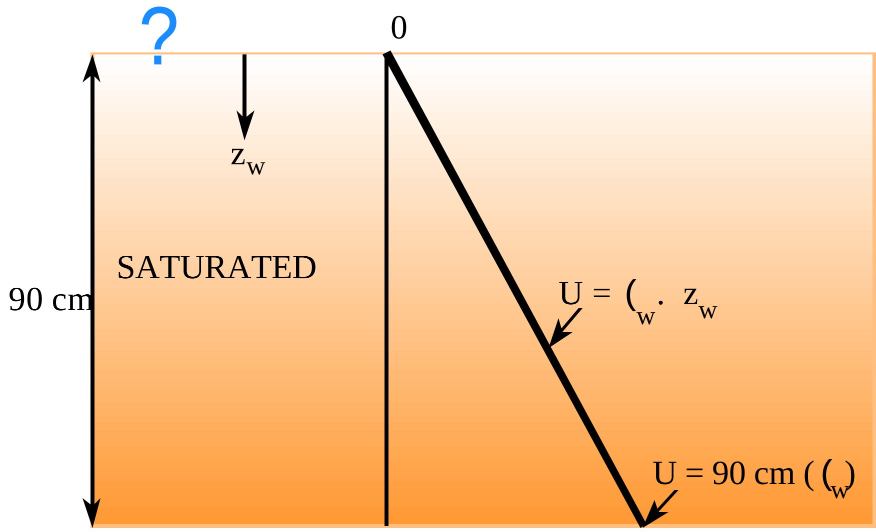
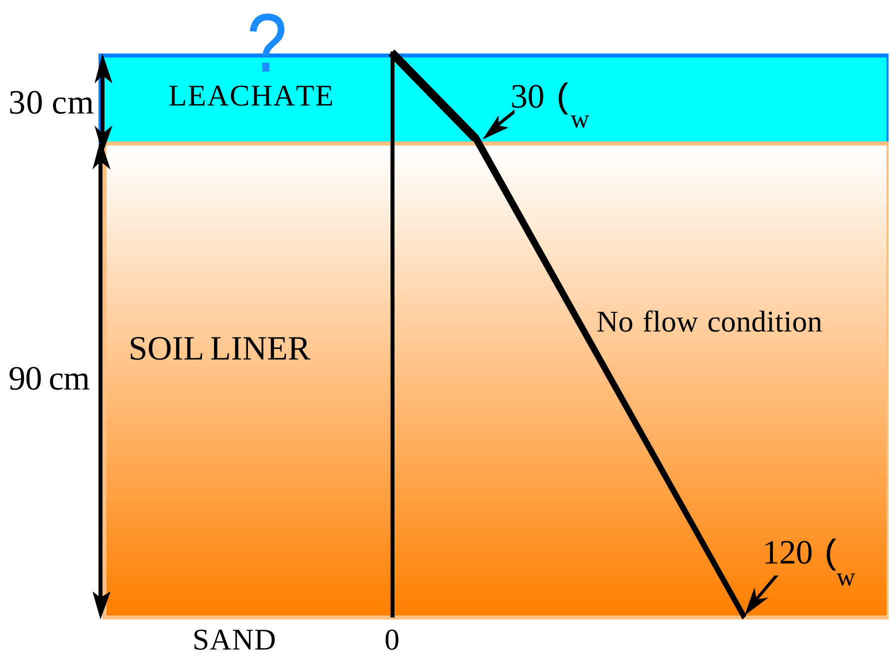


Figure 4.13 Depth versus moisture content variations for using SOILINER and COMPBAR programs (after 1 year)



(a) For a saturated soil



(b) For the soil profile with 30 cm leachate placed on top of it

Figure 4.14 The variations of total stress with depth

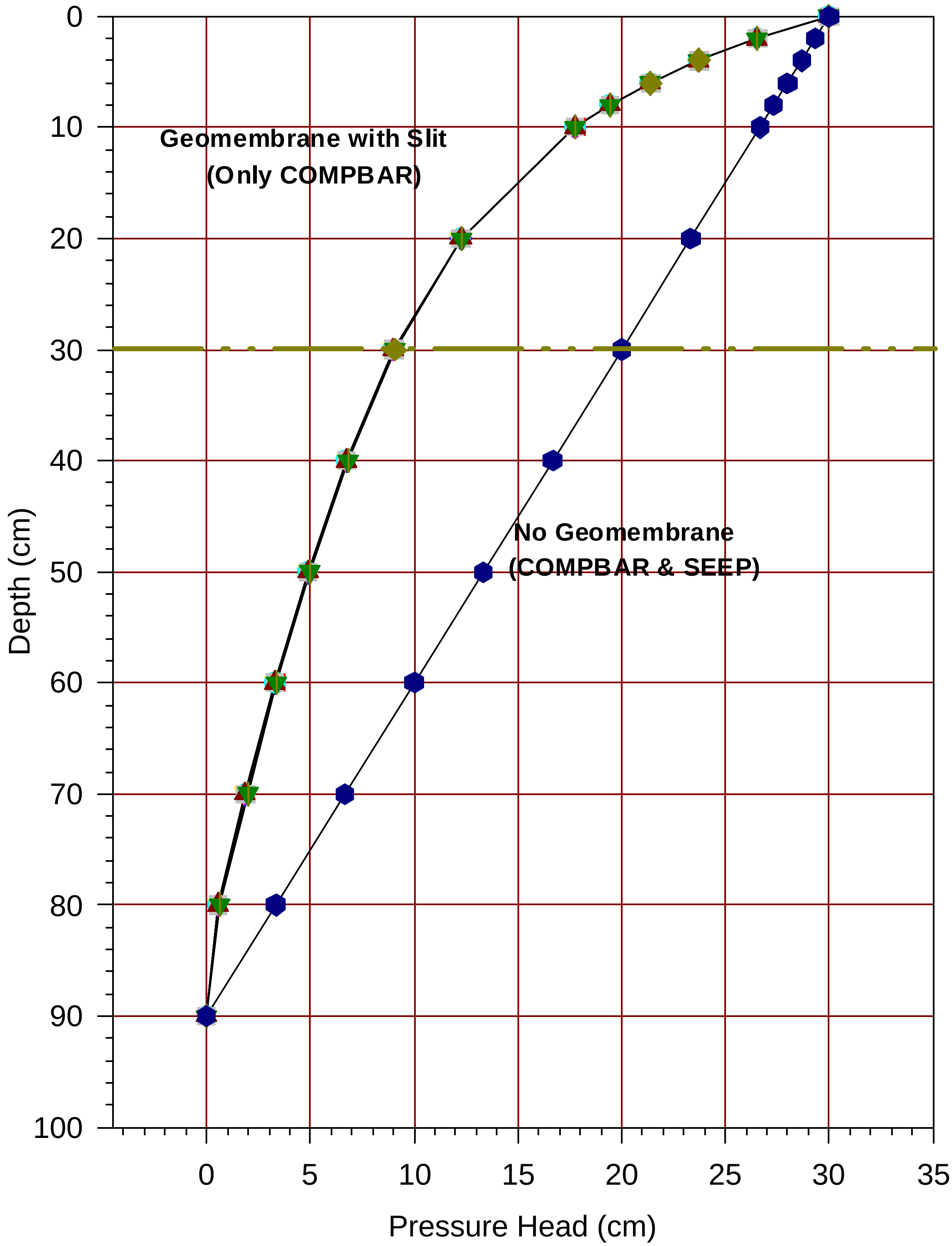


Figure 4.15 Variation of pressure head with depth below the slit with or without slit on geomembrane at saturated soil

Another case shown in Figure 4.15 was evaluated. The soil liner was saturated, and a damaged geomembrane was placed on the top of it. The variations of pressure head with depth are nonlinear and the values are smaller than those for no geomembrane case. For example, at the depth of 30 cm, pressure head for the no geomembrane case was 20 cm, and for the case with the damaged geomembrane was 9 cm. For the case of no geomembrane on the top of the soil profile, there is linear decrease in head. However, for the case of a geomembrane with a slit on the top of the soil liner, there is non-linear decrease in soil because the soil liner is unsaturated which impedes flow.

Consequently, SEEP and COMPBAR give the same results for the no geomembrane case. Thus, COMPBAR appears to result in correct values for saturated flow conditions. With the geomembrane on top of the soiliner case, the head loss rate is greater than for the no geomembrane case.

4.5 Stability of COMPBAR Computer Program

Three different modeling parameters were used to study stability of the COMPBAR program for the case with a slit in the geomembrane. These parameters included the head of the leachate impoundment, the value of the saturated hydraulic conductivity, and the size of the grid. The variables and conditions of the soil profile are given in Figure 4.16. The results of these evaluations are shown in Figures 4.17 through 4.25.

The effect of head of the impounded leachate was evaluated first. The head of the leachate on the top of the geomembrane was varied as 3 cm, 30 cm, and 300 cm in Figures 4.17, 4.18, and 4.19. The value of the pressure head at the top of the clay barrier increases as the impoundment head increases, and decreases as the impoundment head decreases.

As time progresses the soil approaches full saturation (i. e., towards saturated levels of 0.40) as the wetting leachate front moves down into the soil. Volumetric moisture content (θ) is equal to the saturated level (θ_s) at the top of the geomembrane and decreases nonlinearly downward through the soil profile for each leachate head. As time increases, volumetric moisture content

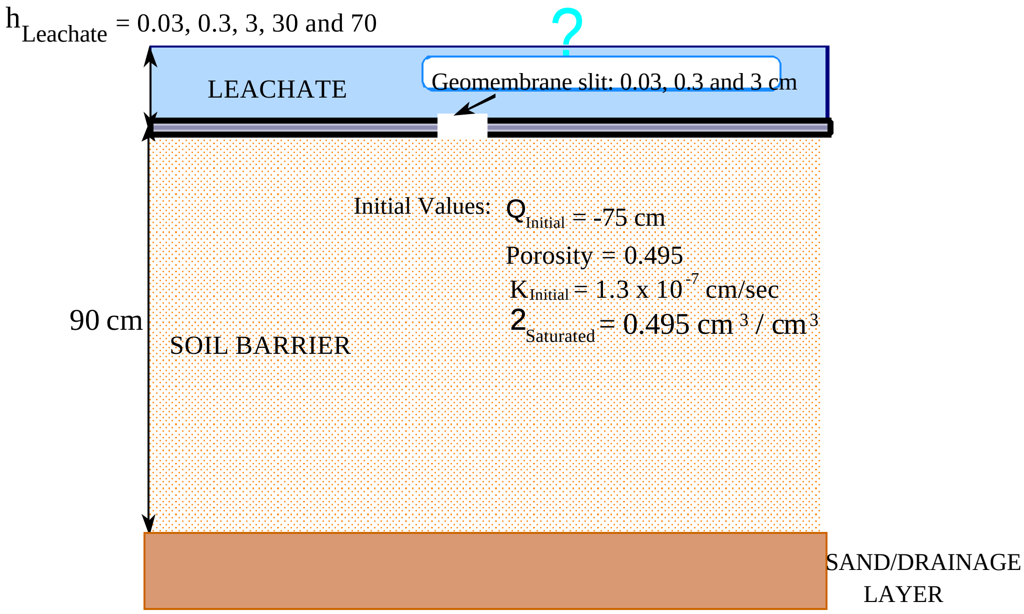


Figure 4.16 The variables and conditions of the soil profile

(and degree of saturation) increases as the wetting area progresses down into the soil. The degree of saturation (S) can be defined as the ratio of the volume of water (V_w) to the volume of voids (V_v) , $S = V_{ow}/V_v$. Figure 4.17 shows that after one year for 3 cm of leachate head the soil profile is saturated (θ = 0. 40) to a depth of 10 cm, and then θ value sharply decreases to about 0. 013 at depths between 28 cm and 85 cm. As time increases, the saturation front is progressing downward through the soil profile reaching full saturation. For example, after eight years, at a depth of 65 cm, θ has a value of 0. 367 and continues to increase towards full saturation as indicated by the "19 years" curve. Figures 4.18 and 4.19, with 30 cm and 300 cm leachate head, respectively, show that the wetting front progresses faster down into the soil profile with increasing leachate head. After 19 years, full saturation reached a depth of about 50 cm. For the 30 cm of leachate head (Figure 1.1) after 20 years, the clay barrier is saturated up to a depth of about 80 cm. For the 300 cm leachate head, it was observed that after 1 year most

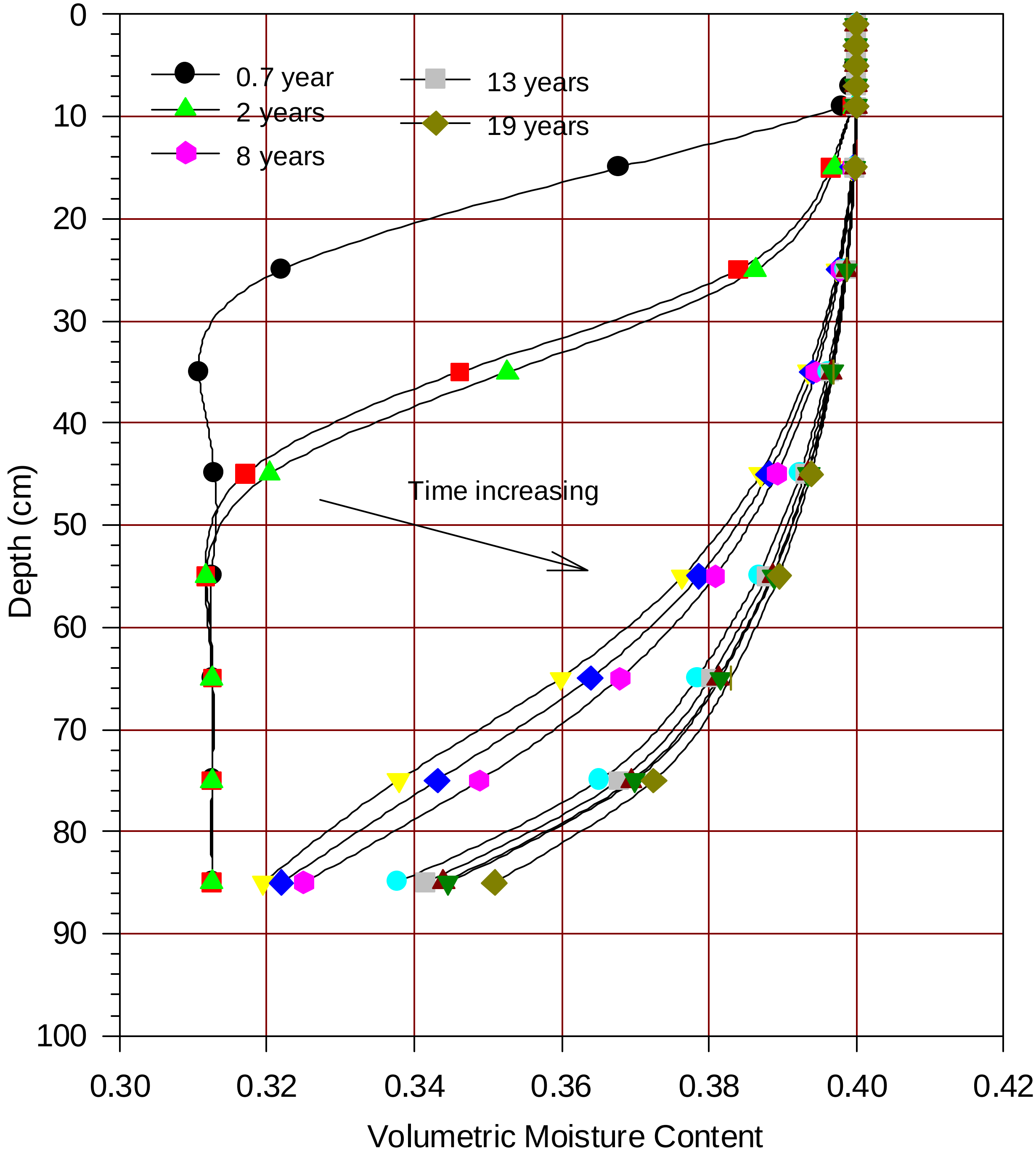


Figure 4.17 Volumetric moisture content variation with 3 cm leachate height

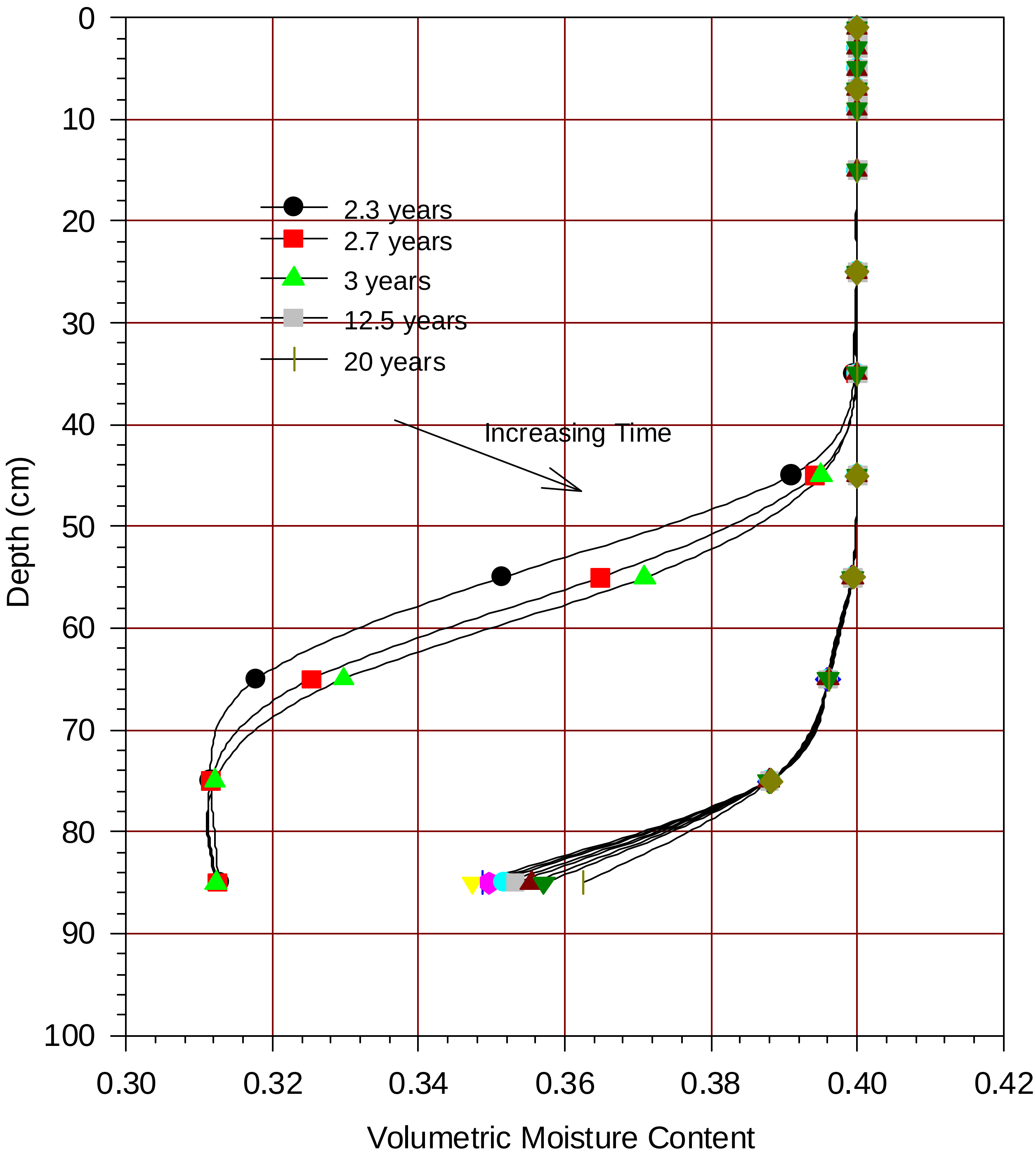


Figure 4.18 Volumetric moisture content variation with 30 cm leachate height

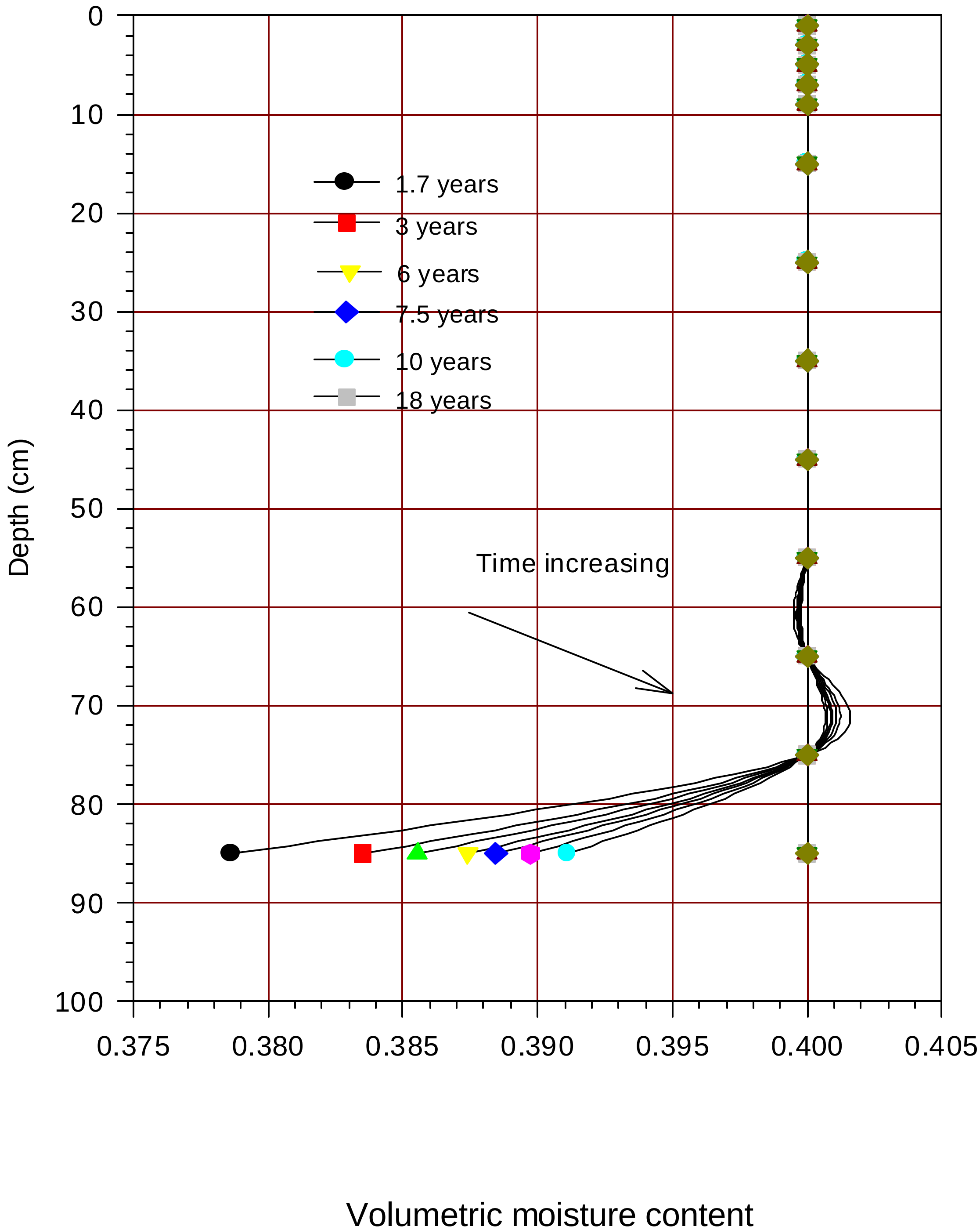


Figure 4.19 Volumetric moisture content variation with 300 cm leachate head

of the soil profile was saturated, which indicates that a higher depth of impoundment yields faster wetting front movements as expected. Obviously, the head of pounded leachate on the soil liner strongly affects the rate of movements of the saturated wetting front.

The next step of the stability analyses was to study the effect of hydraulic conductivity variation in the soil barrier. The saturated hydraulic conductivity was varied as follows: $1x10^{-4}$ cm/sec, $1x10^{-7}$ cm/sec and $1x10^{-10}$ cm/sec as shown in Figure 4.20 through 4.22. It can be seen from these figures that the wetting front reaches the collection layer after one year. If the soil barrier has a very high hydraulic conductivity value as in this case, the wetting front moves rapidly down in the soil as compared to the case with a very low hydraulic conductivity soil. For a normal case of hydraulic conductivity, the behavior is reasonable where the soil is saturated up to the depth of 45 cm. The part of the soil profile which is under that depth still has the initial volumetric moisture content value.

The final stability examination of COMPBAR evaluated the effect of variation of grid sizes of the finite element mesh used for simulation (Figure 4.23 to 4.25). A grid size of 200 cm was employed to conduct all the analyses in this study. In addition, two different grid sizes were also used as100 cm and 300 cm. It can be seen from all obtained figures for all time increment that the volumetric moisture content versus depth trends are similar for the three different grid sizes, 100 cm, 300 cm, and 600 cm. Therefore, the 200 cm grid size was used throughout this study.

4.6 Results of Parametric Analyses

These examples simulate different conditions of soil-geomembrane contact, slit size, and different leachate head for a given and hydraulic conductivity of the soil barrier. Twenty variations were studied, including Examples 1 and 2, which are shown in Tables 4.9 and Figure 4.16. The overall flow rate values as defined by Giroud and Bonaparte (1989 a, b) (see Equations 2.1 and 2.2 in Chapter 2) can be observed from this table. The selected outputs of several examples were also presented in tables and figures in Section 4.2 in this chapter.

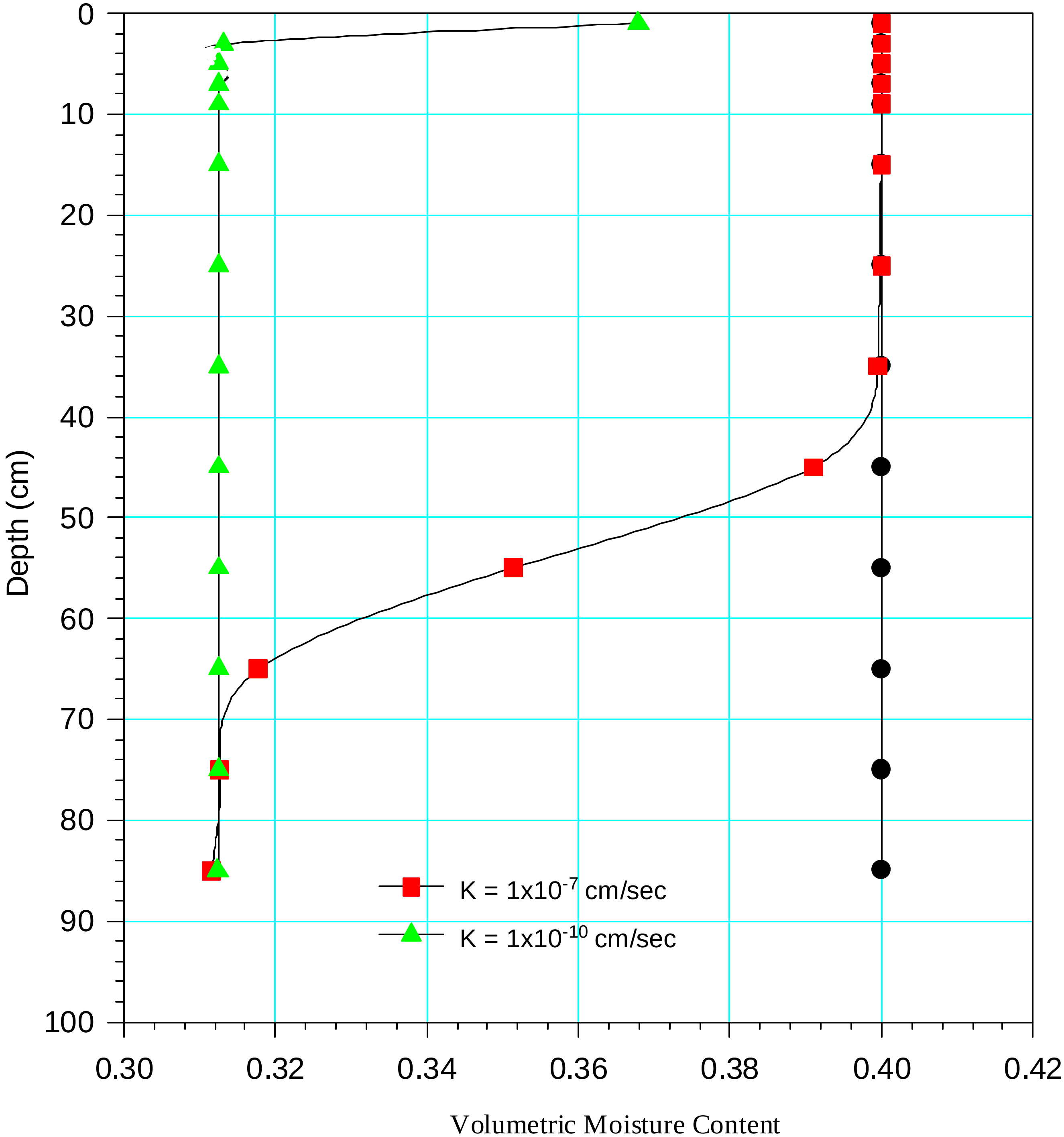


Figure 4.20 Change of volumetric moisture content with depth below the slit after 1 year

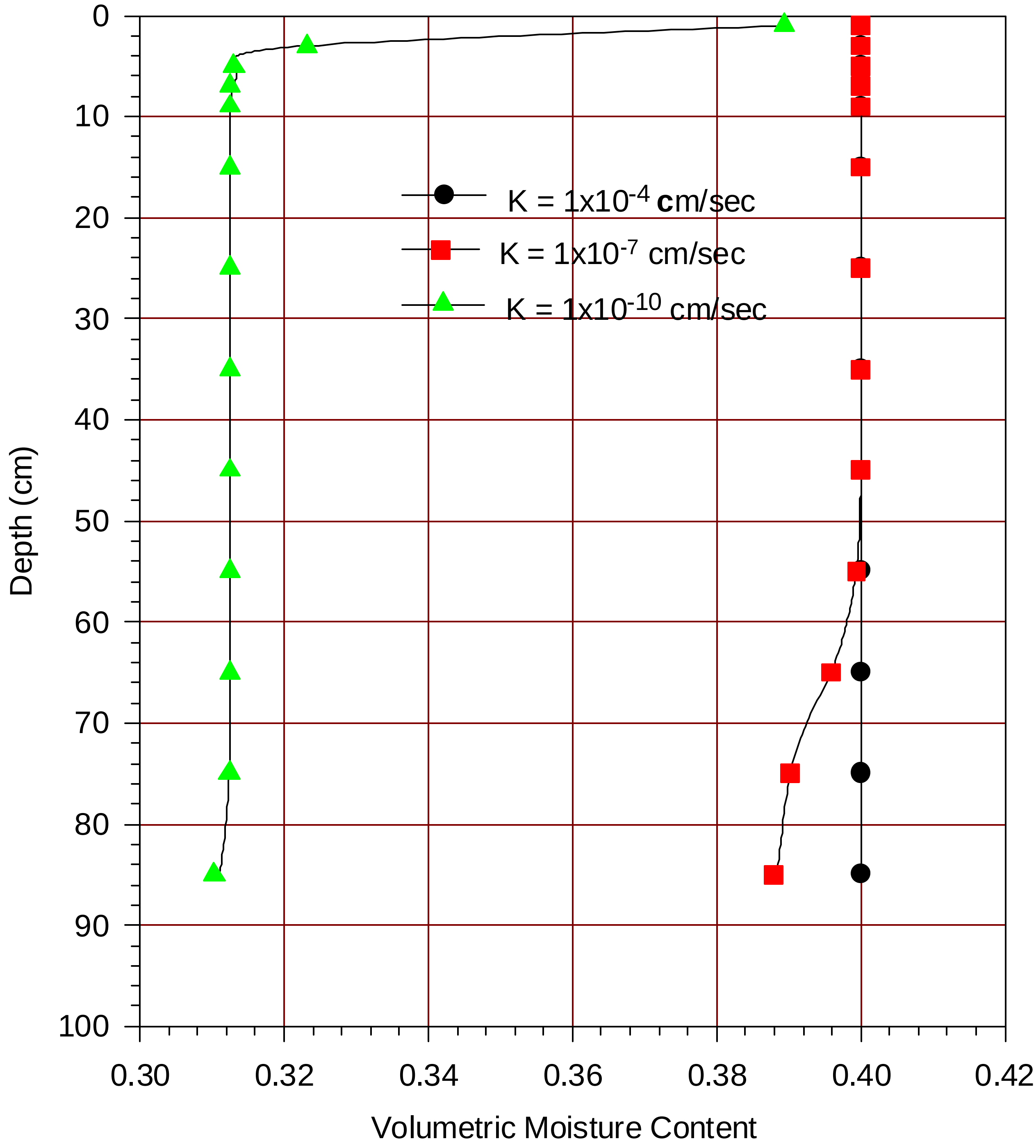


Figure 4.21 Change of volumetric moisture content with depth below the slit after 8 years

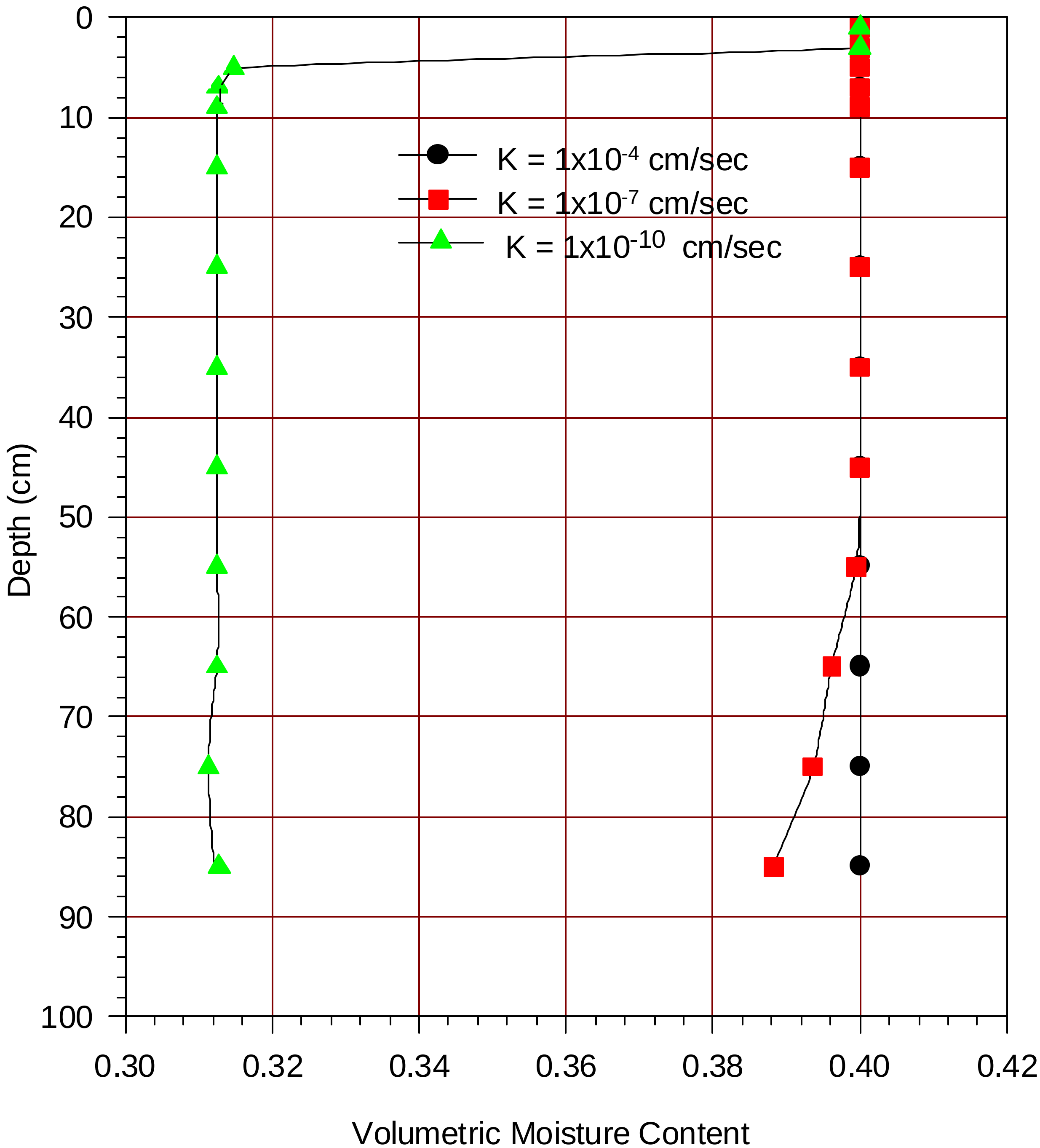


Figure 4.22 Change of volumetric moisture content with depth below the slit after 21 years

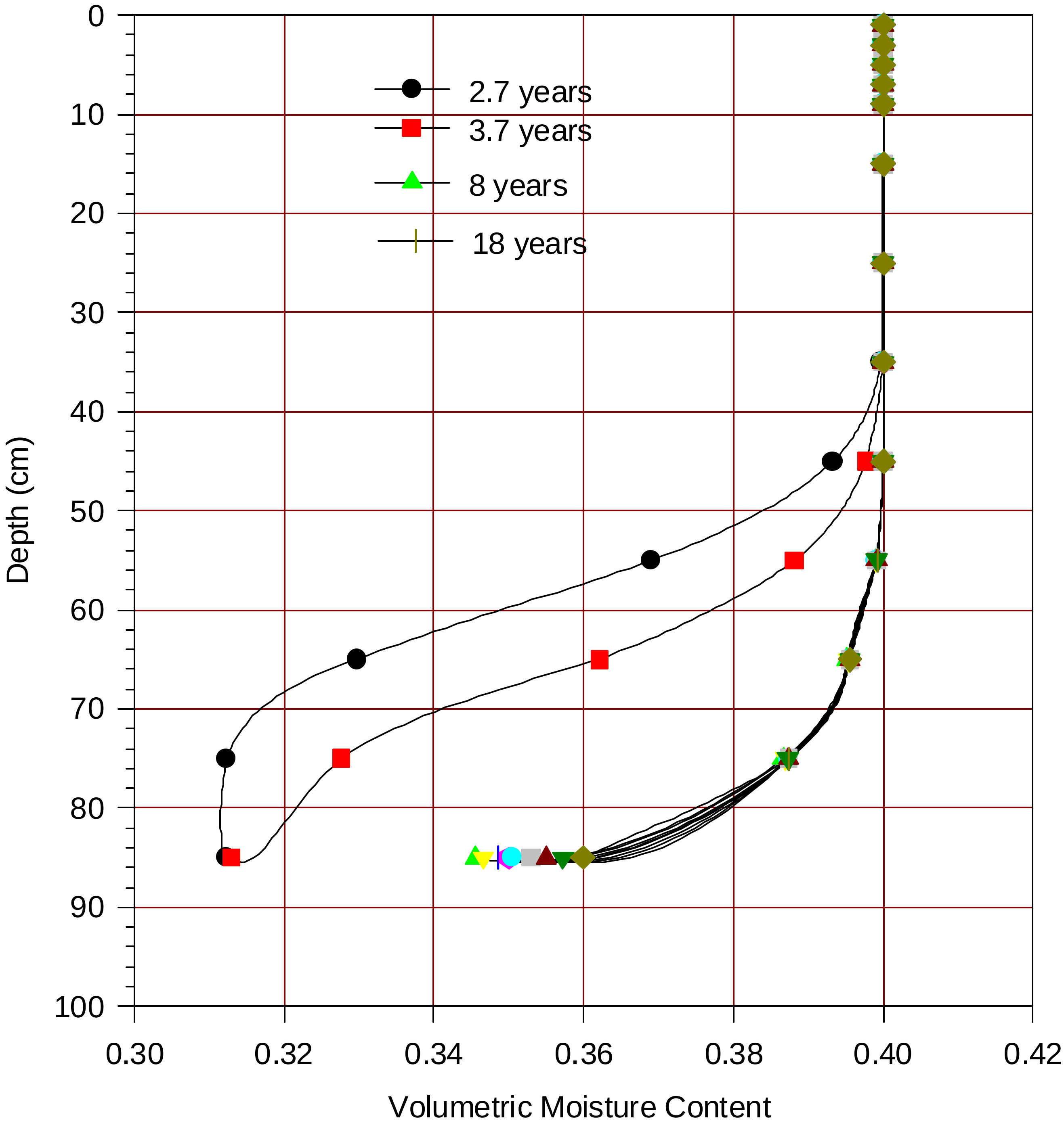


Figure 4.23 Volumetric moisture content variation by using 100 cm grid size

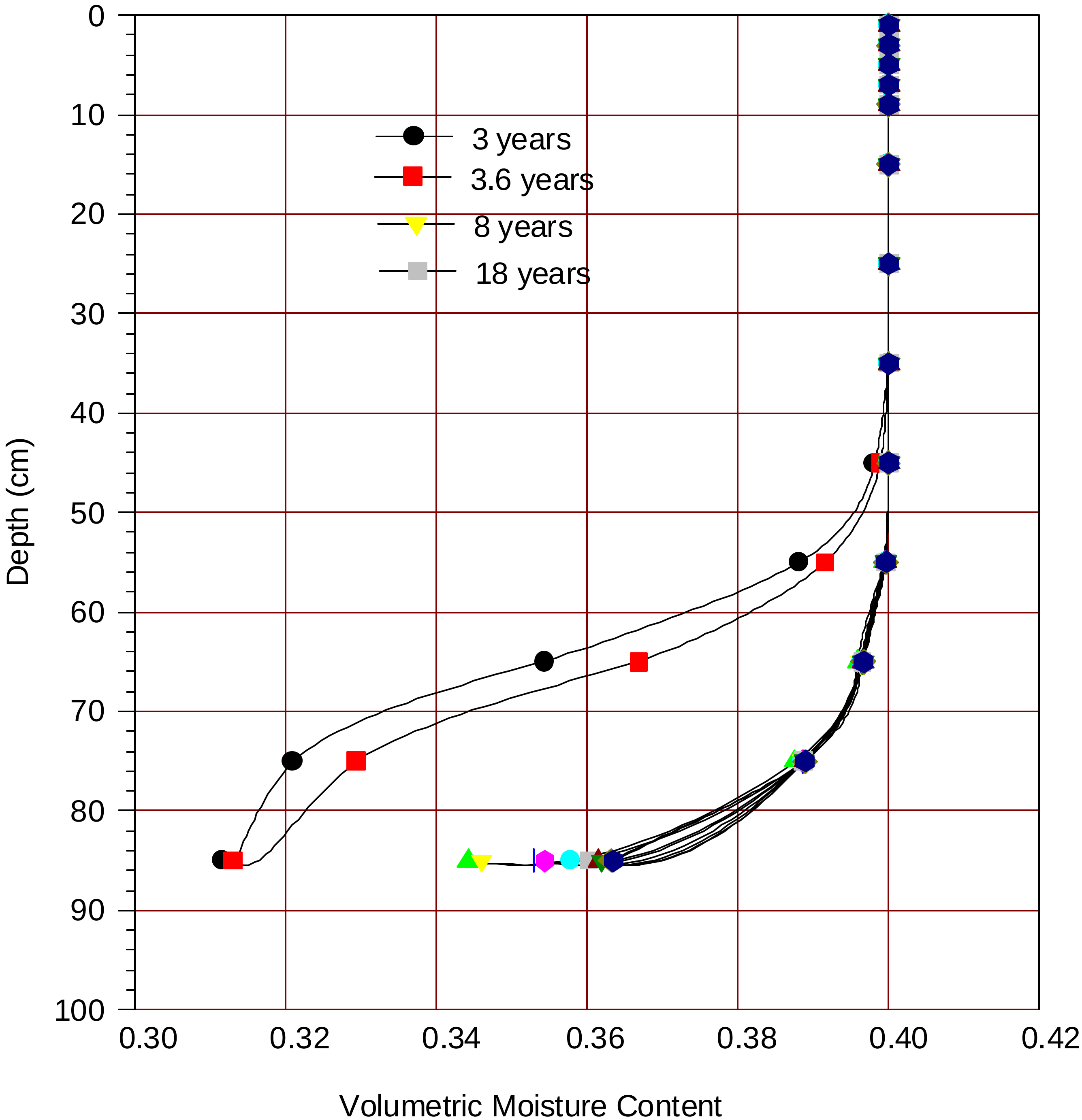


Figure 4.24 Volumetric moisture content variation by using 300 cm grid size

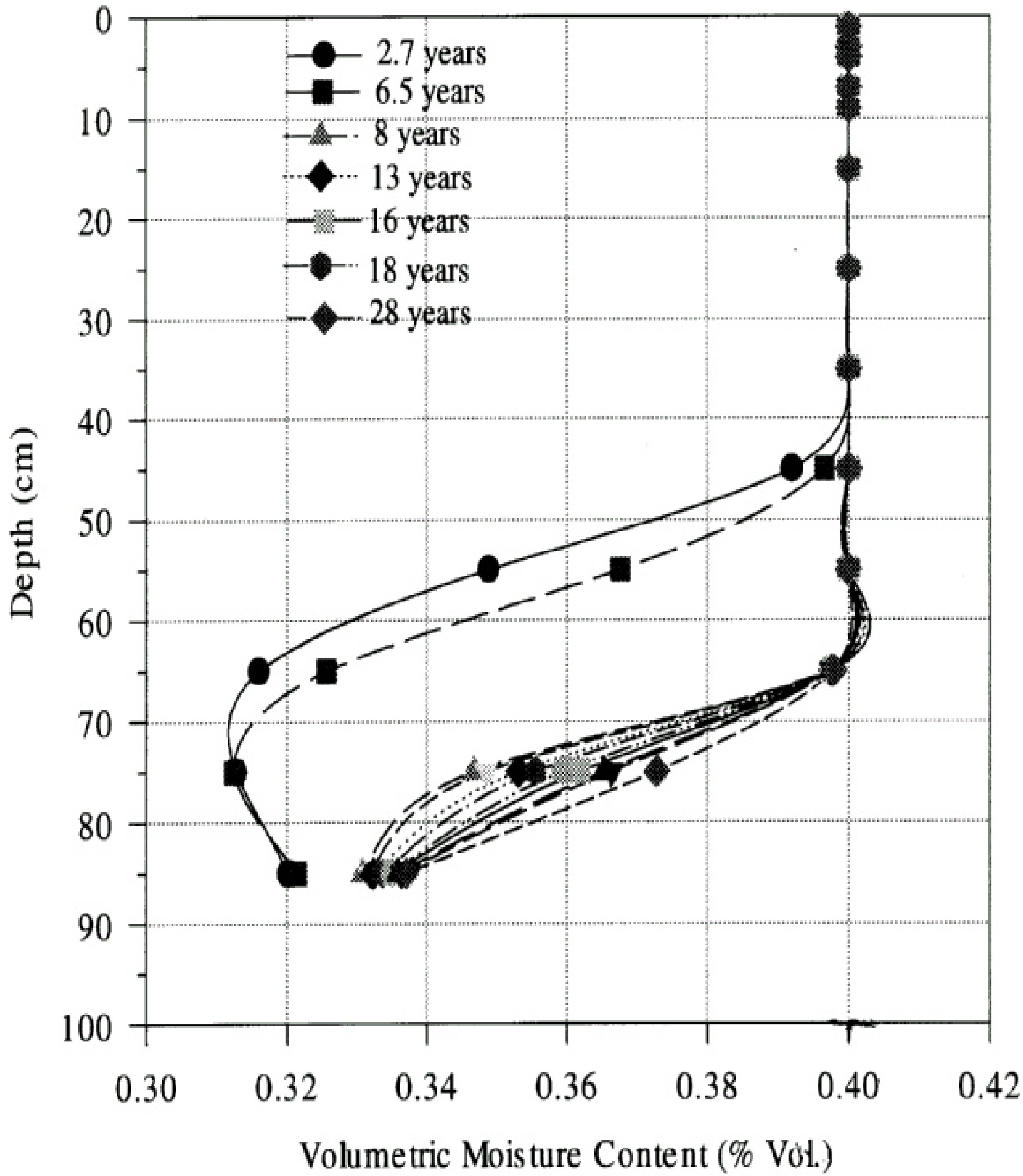


Figure 4.25 Volumetric moisture content variation using a 600 cm grid size

The solution obtained from the iterative procedure from the finite element program provides the values of pressure head (ψ) at the nodes. Volumetric moisture content (θ), hydraulic conductivity (K), and flow rate (q) were obtained from equations that are functions of pressure head. The comparison of the different cases that are given in Table 4.9 will be based on pressure head values. The comparison of pressure head under different conditions of composite barrier are shown in Figure 4.26 through 4.34.

It can be seen in Figure 4.26 that pressure head values for good contact and poor soil-geomembrane contact are similar after one year. There is a maximum value close to the geomembrane and decreases nonlinearly to the minimum value in both cases. From the same figure, after one year and at a depth of 40 cm, pressure head value is –36 cm for poor contact and –27 cm for good contact. This indicates that, at the same location, the moisture content levels for the poor contact were slightly higher than those for good contact. For different time periods, for example, after 10 years and at a depth of 60 cm good contact conditions produce –4 cm and –1 cm of the pressure head, respectively. The slight difference in pressure head between a good soil-geomembrane contact and poor contact is significant in determining the effectiveness/longevity of the barrier system. A poor soil-geomembrane contact condition results in slightly higher pressure head and wetting front movement. With a good soil-geomembrane contact condition, the geomembrane will maintain the barrier system effectiveness for a longer period of time.

The results of Figures 4.26 and 4.27 show a slight difference in pressure head comparing poor contact and good soil-geomembrane contact conditions. In Figure 4.26, for example, after one year at a depth of 30 cm poor and good contact conditions behave the same. Between a depth of 30 cm and 50 cm, poor soil-geomembrane contact conditions result in a slightly higher pressure head, while from 50 to 90 cm of depth, the pressure head is the same. After 10 years, the pressure head values behave the same at all depths. Figure 4.27 compares poor and good contact conditions with 70 cm leachate head. The pressure head difference after four years becomes more pronounced because of the higher leachate head resulting driving force. For instance, for good soil-geomembrane contact conditions, after a depth of 50 cm, the pressure head is less than that of the poor soil-geomembrane contact conditions. After 4 years at depth of

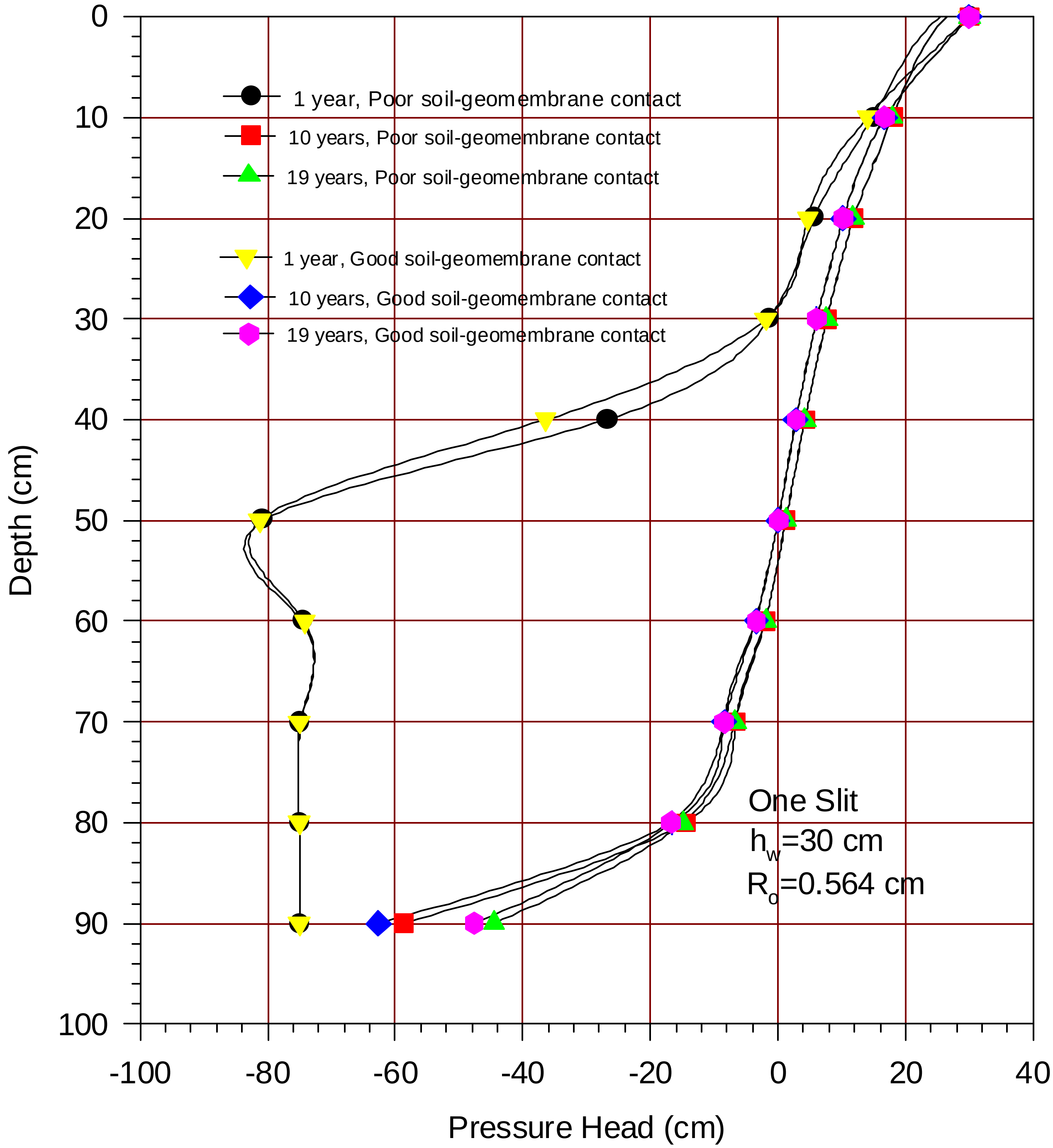


Figure 4.26 Comparison of poor and good soil-geomembrane contact for one slit and 30 cm of leachate height

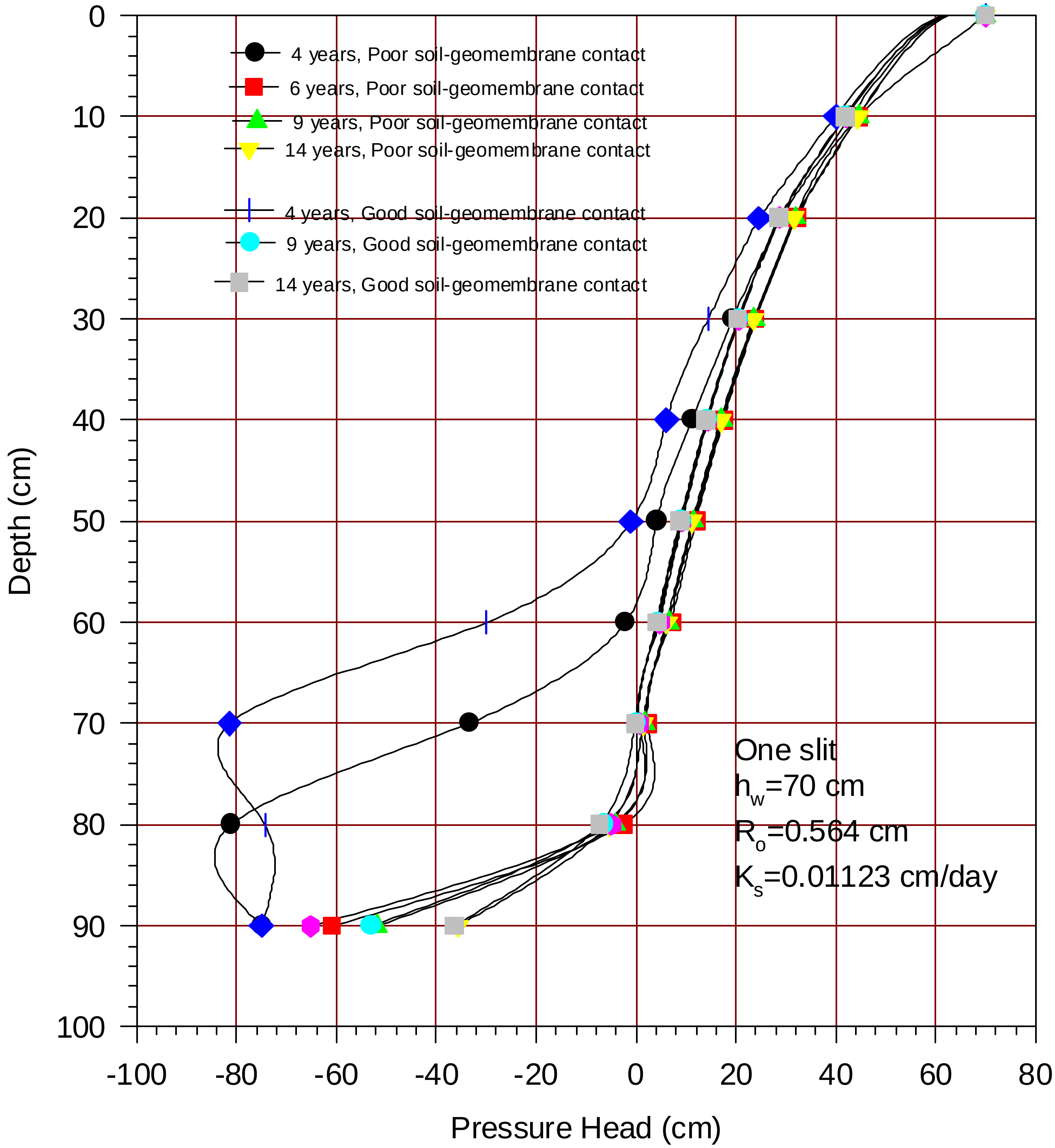


Figure 4.27 Comparison of the poor and good soil-geomembrane contact for one slit and 70 cm of leachate height

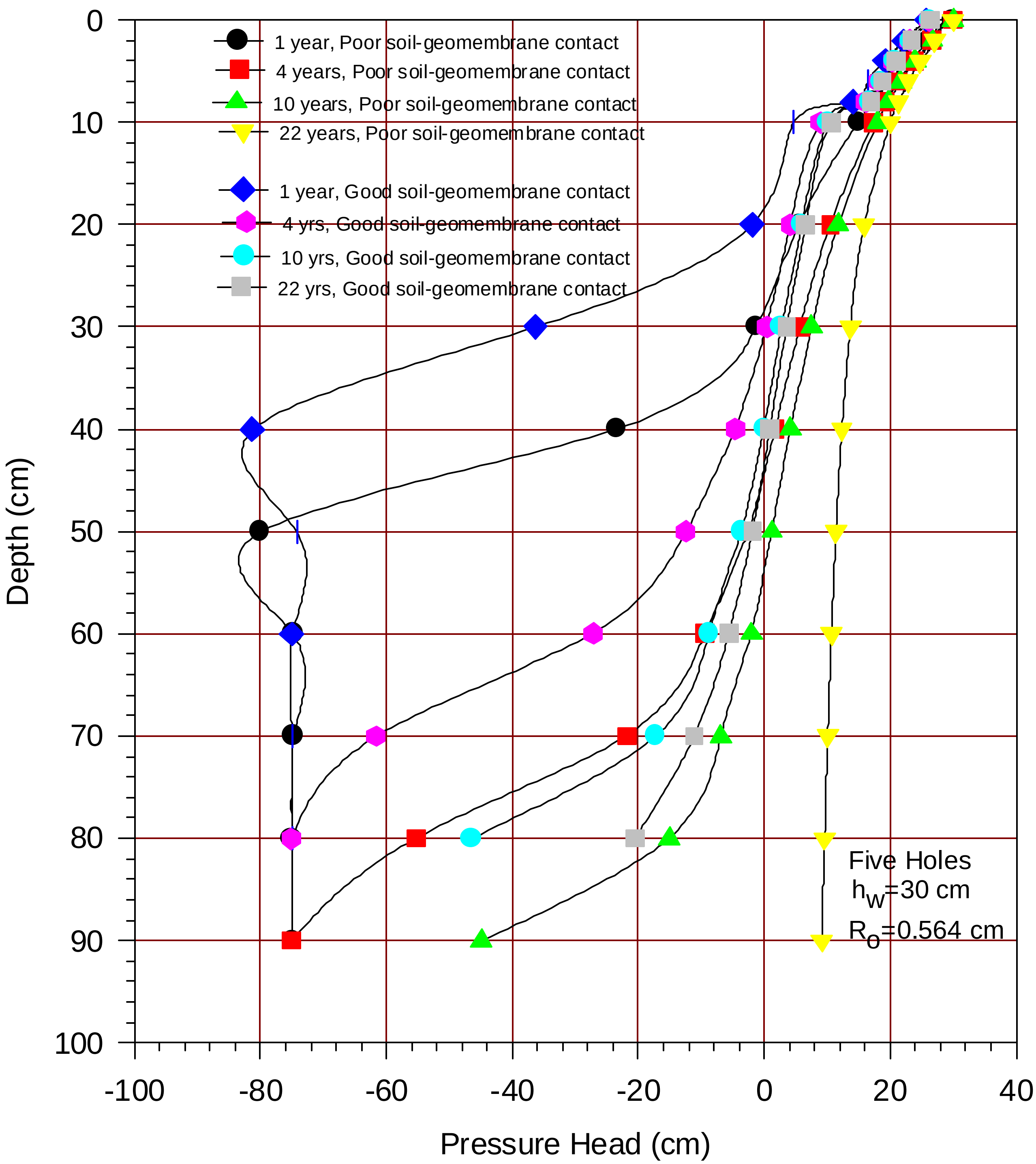


Figure 4.28 Comparison of pressure head for poor and good soil-geomembrane contact for five slits and 30 cm of leachate height

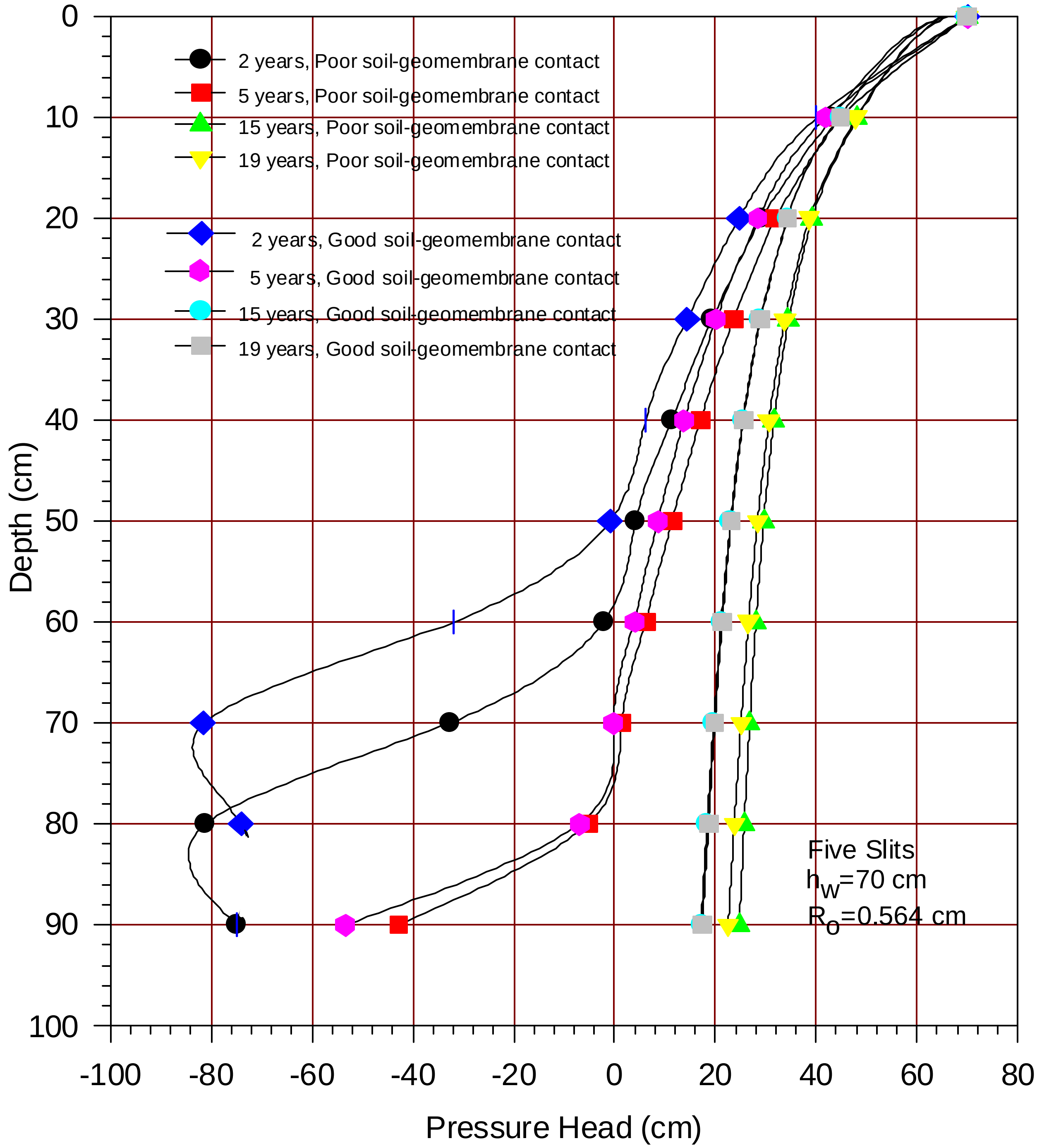


Figure 4.29 Comparison of pressure head of poor and good soil-geomembrane contact for five slits and 70 cm of leadhate height

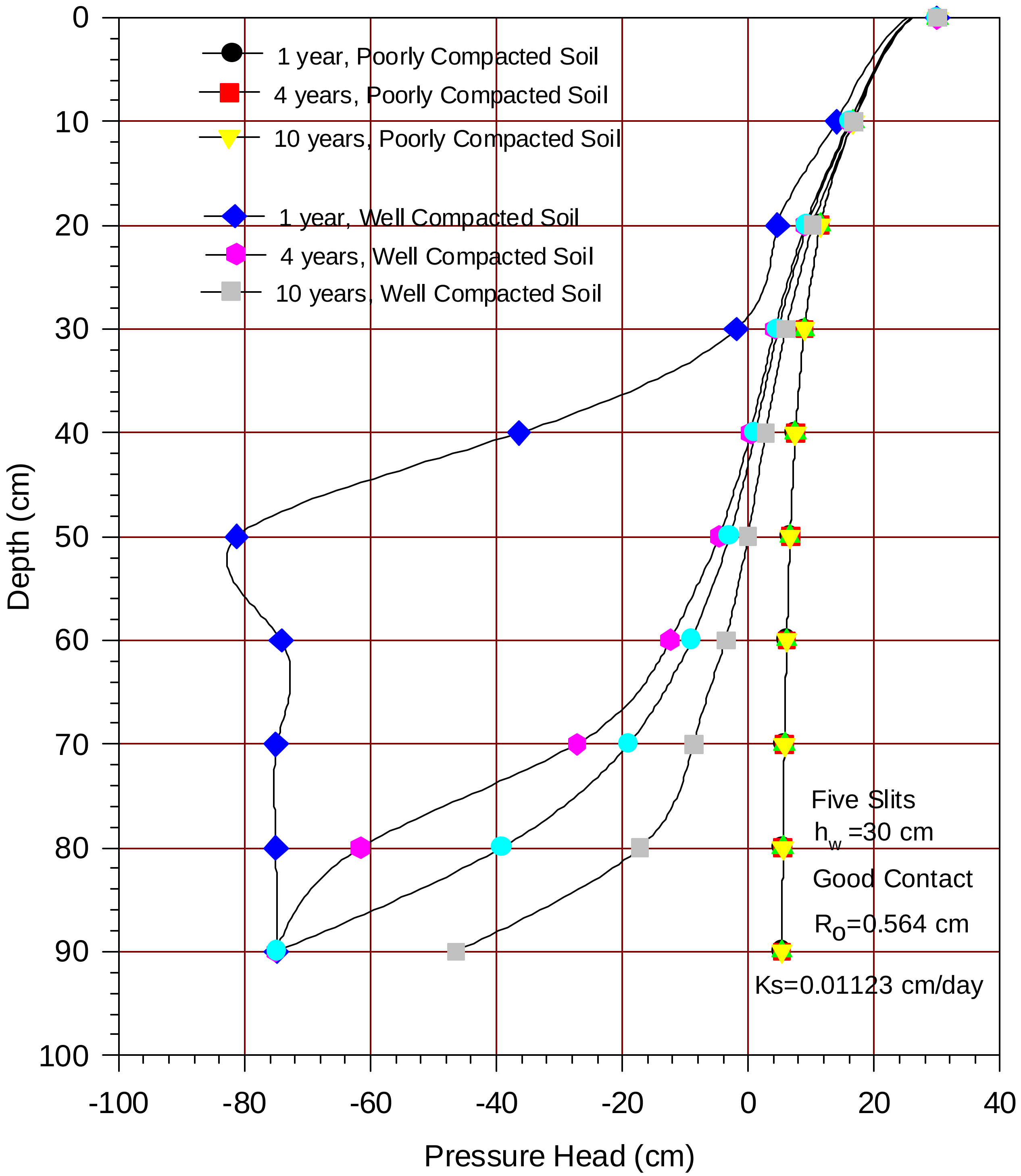


Figure 4.30 Comparison of pressure head variations for porly compacted and well compacted soil

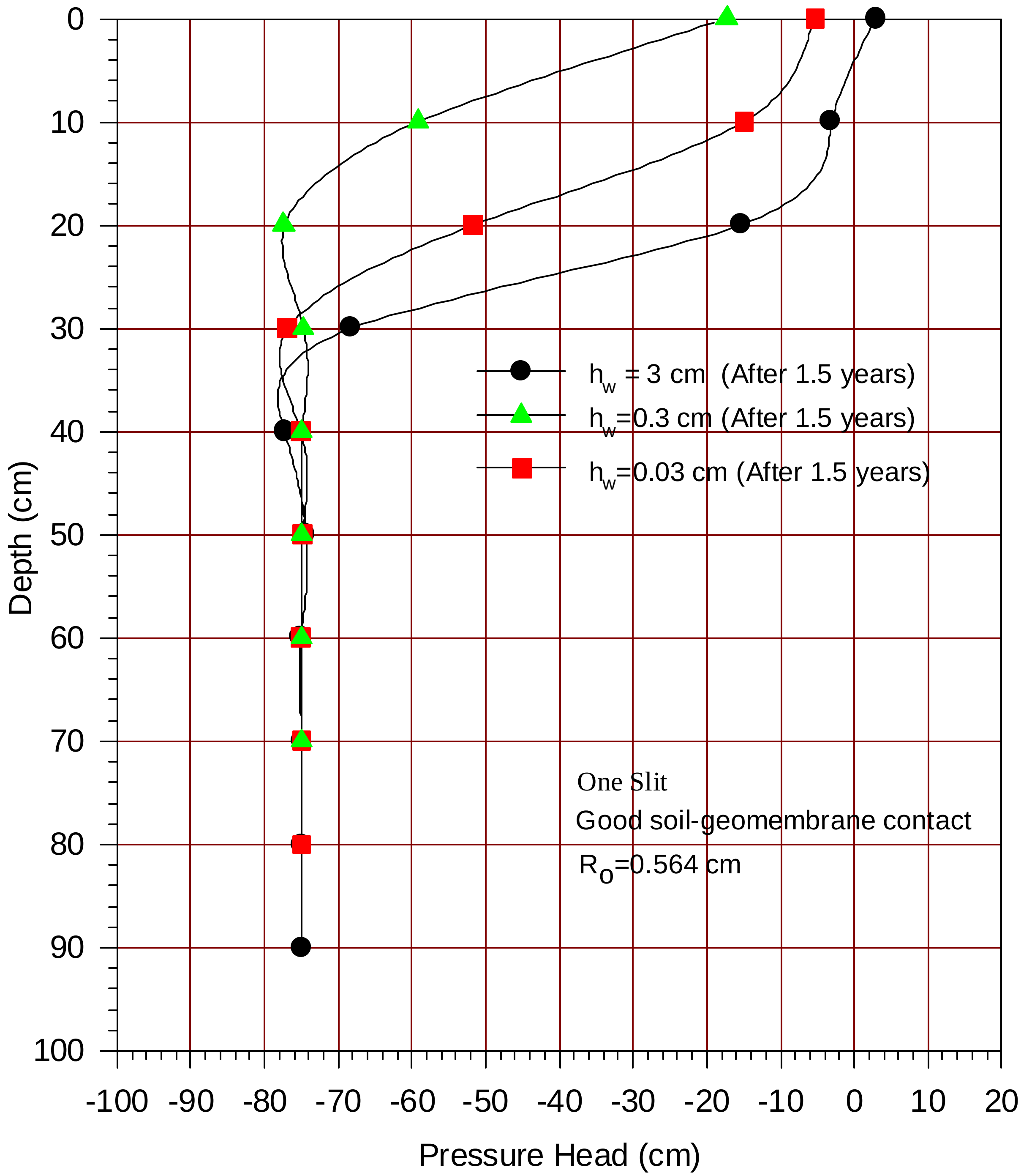


Figure 4.31 The pressure head variations for the 0.03cm, 0.3cm, and 3 cm leachate heights

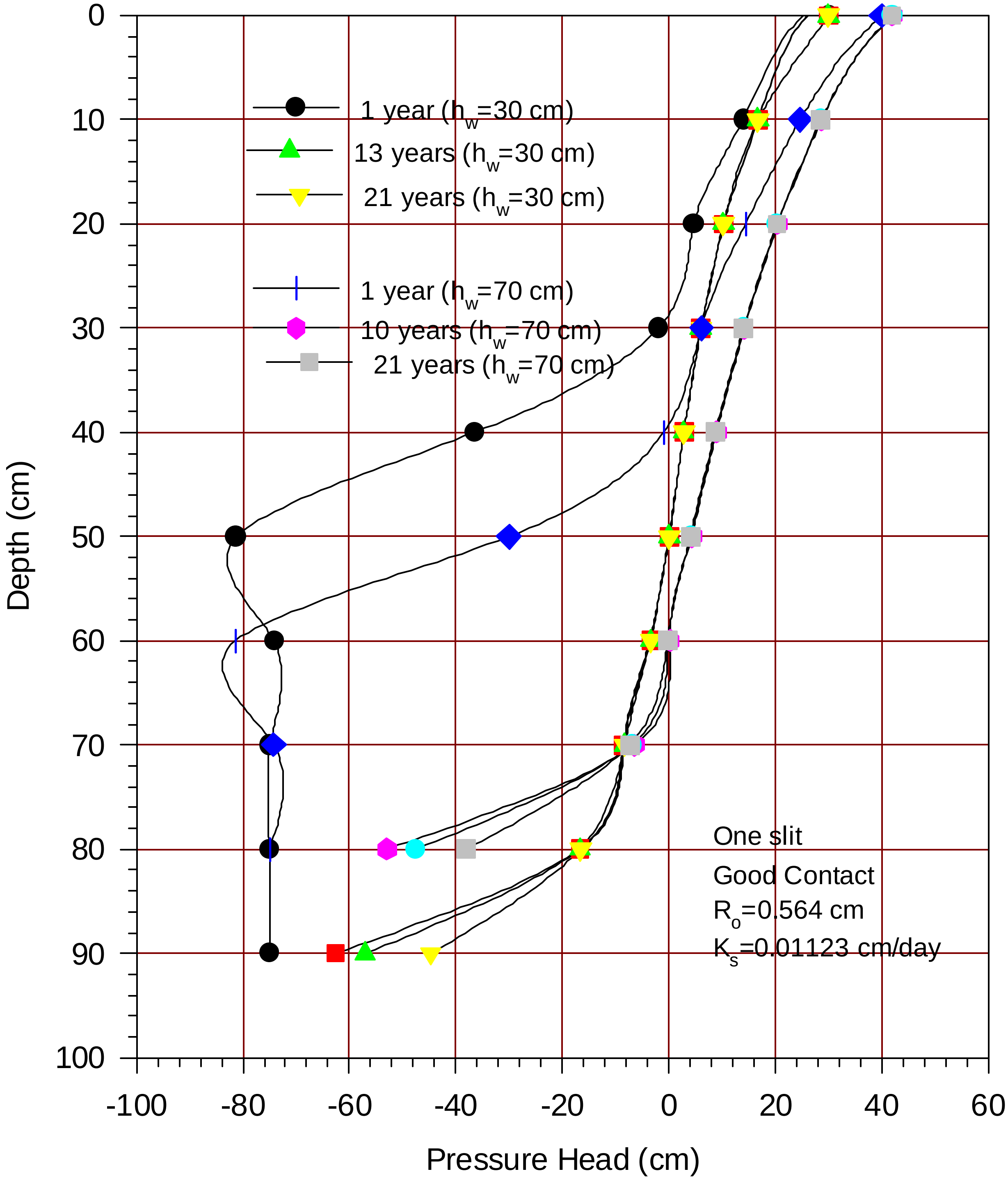


Figure 4.32 Comparison of pressure head with different leachate heights for good contact and one hole.

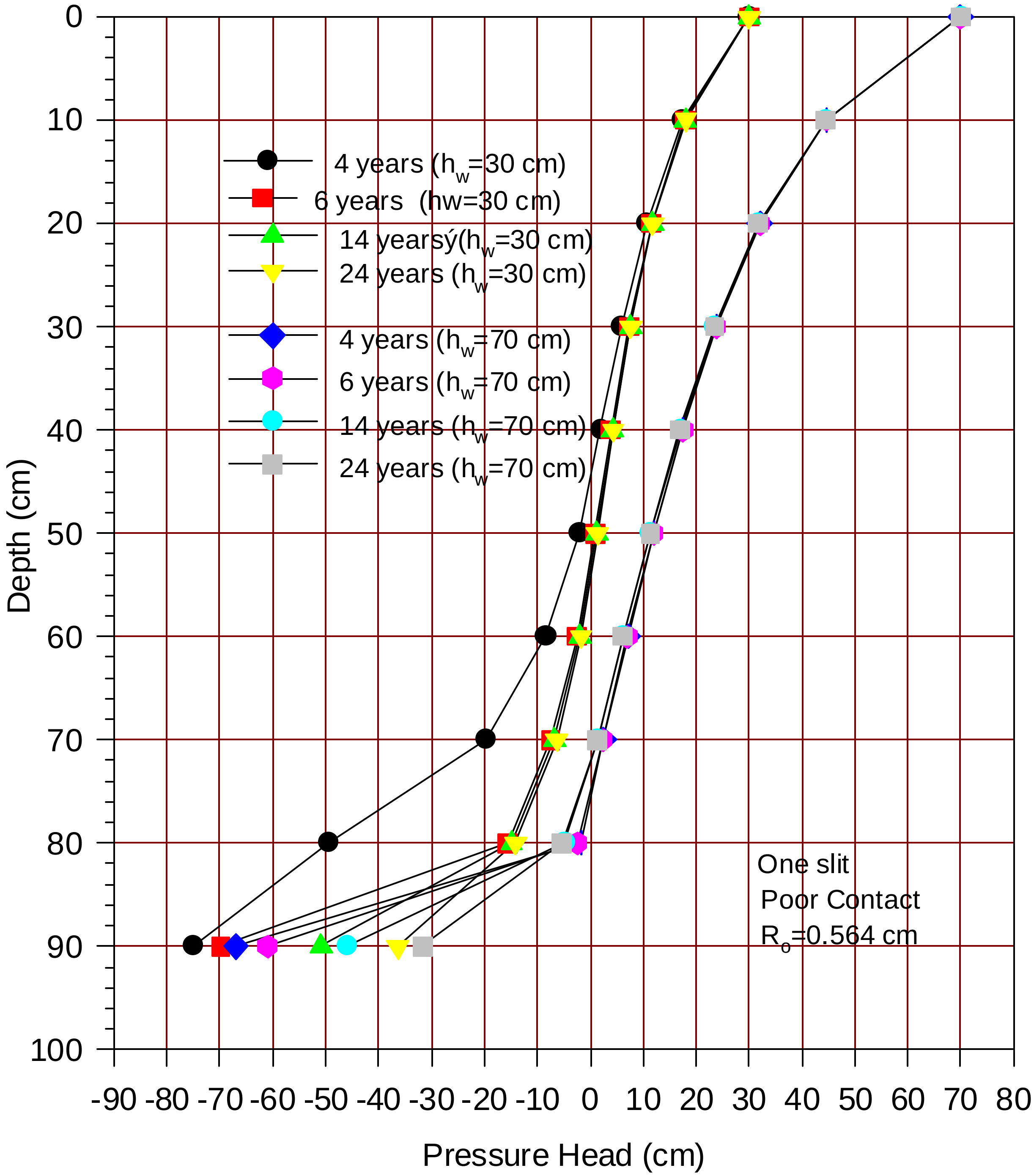


Figure 4.33 Comparison of pressure head with different leachate heights for poor soil-geomembrane contact and one slit

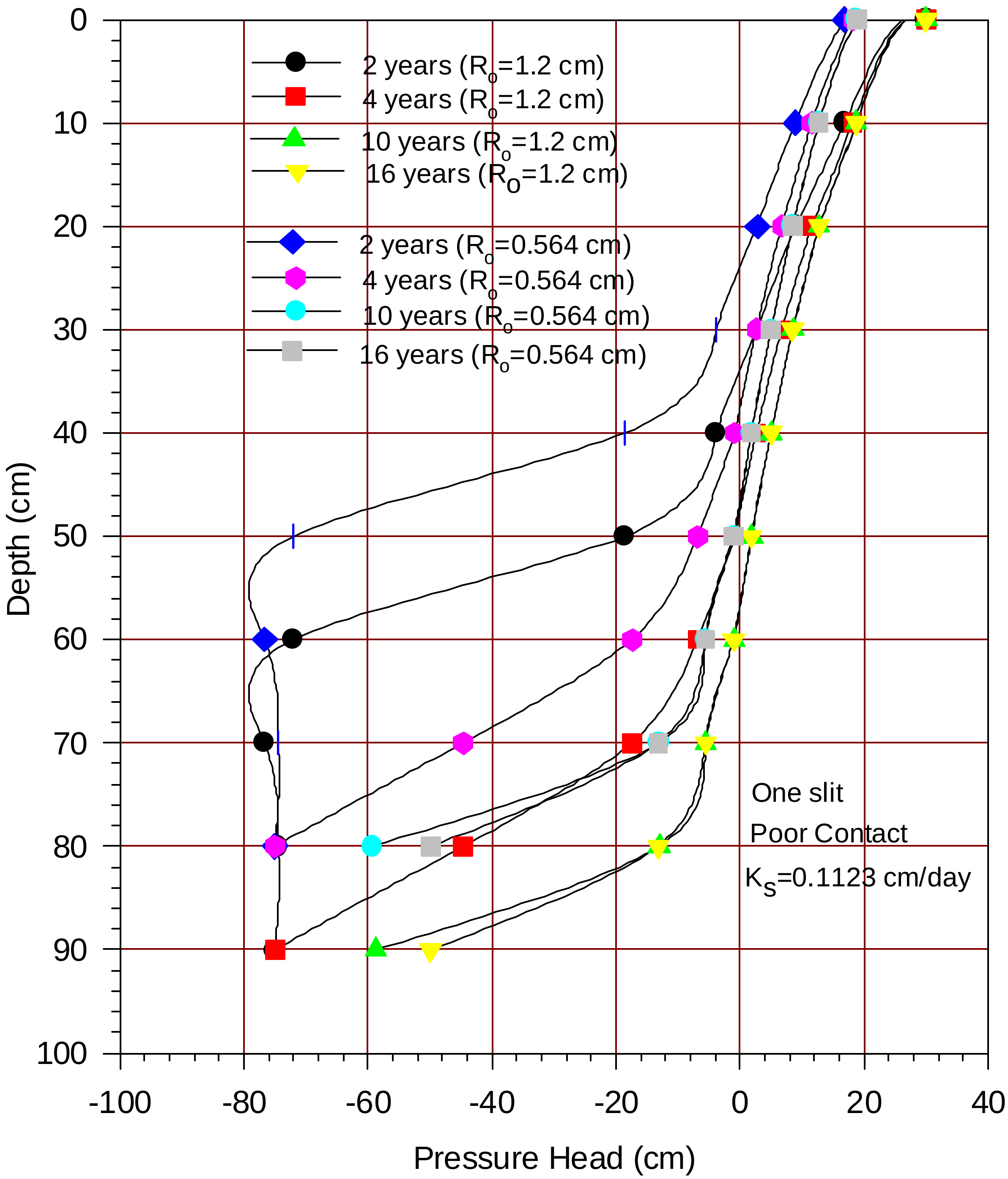


Figure 4.34 Comparison of pressure head with several sizes of slits for one slit and poor soil-geomembrane contact condition

60 cm, pressure head (ψ) values have different values. It is concluded that the wetted front movement of the poor contact condition is a little bit faster than that for the good contact.

Next, it was assumed that the geomembrane had five slits at specified locations (see Figure 3.18). A comparison of the cases of poor and good soil-geomembrane contact was conducted (Figure 4.28). It can be seen that the poor contact condition produces higher values of pressure head, which is an indication of higher values of moisture levels and that the wetting front migrates slower through the soil profile with good soil-geomembrane contact than poor contact conditions. For example, after 4 years at depth of 60 cm, pressure head values are –28 cm for the good contact and –10 cm for poor contact condition. After 22 years, the slit depth of the soil profile is already saturated for the poor contact condition compared to the good contact condition, as indicated by the positive head values.

Similar observations can be also extracted from Figure 4.29. It is shown that the saturation levels are higher in the case with a poor contact than in the case with good contact condition after two years. The pressure head is slightly higher for the poor soil-geomembrane contact conditions, and even more pronounced than in Figure 4.28, due to higher leachate heads.

The COMPBAR program performs another verification of a barrier system by comparing pressure head variations in poorly compacted soil conditions. Figure 4.30 shows that the poorly compacted soil became saturated in a very short time period (as demonstrated by the curve on the right extreme of the figure). After less than one year, the whole soil profile reached saturation, i. e., positive ψ values. In the compacted clay barrier there is gradual movement towards saturation from one to twenty one years. The summary of Figure 4.30 indicates a very short life span of a composite barrier made of loose soil, and also suggests that the degree of compaction is an important point in the construction and performance of the barrier.

The effect of leachate head was also studied. Five different leachate heads on top of the geomembrane have been (0.03 cm, 0.3 cm, 3 cm, 30cm, and 70 cm) were utilized. In Figure 4.31, the three lowest leachate head are compared. Pressure head varies considerably for each leachate head up to a depth of 30 cm. Below that depth, pressure head remains the same for all leachate heads.

Leachate heads of 30 cm and 70 cm are compared in Figure 4.32 with good soil-geomembrane contact conditions. After one year, the pressure head is higher with the higher leachate head (70 cm) from a depth between 30 cm to 50 cm. After a depth of 50 cm to 60 cm the pressure head is similar for both leachate heads. At depths between 30 cm to 50 cm the higher pressure, head values for 70 cm of leachate head, as opposed to the values for 30 cm of leachate, indicate a faster wetting front movement. Figure 4.33 shows results for the same leachate heads as in Figure 4.32. However, it compares pressure head values for the two heads under poor soil-geomembrane contact conditions. It can be seen from Figure 4.33 that the pressure head curves for both leachate heads from the top of the soil to the bottom are almost parallel to each other, demonstrating rapid wetting front movements. When comparing these results with good soil-geomembrane contact conditions (as shown in Figure 4.32), the wetting front progresses more slowly under good soil-geomembrane contact conditions, thus resulting in lower pressure heads. Consequently, the good soil-geomembrane contact condition becomes an important factor in determining the effectiveness and the longevity of a composite barrier system.

The top of the clay layer directly beneath the geomembrane has higher pressure head values for 70 cm of leachate head than for the 30 cm of leachate head in Figure 4.33. But the lines start to approach parallel position moving away from the boundary conditions. After 4 years, 30 cm of leachate head produces lower negative pressure head (ψ) than that of the 70 cm of leachate head. For instance, at the depth of 80 cm ψ = -50 cm for 30 cm of leachate head, and ψ = -5 cm for 70 cm of leachate head at the same depth. This comparison indicates that 70 cm of leachate head produces higher moisture content level and faster wetting front propagation. For a time interval after of 22 years, for 70 cm of leachate head, the soil over the slit depth became saturated. For 30 cm of leachate head, the soil became saturated only to a depth of 70 cm. In conclusion, these figures indicate that the leachate head does affect the wetting front propagation pressure head values and moisture content levels at certain locations. Generally, the higher leachate head on top of the geomembrane, the faster the wetting front moves, and the higher the moisture content are levels for the same time and the same location.

Part of the parametric comparison study has been based on the size of the slit. The small sizes of slit considered were R_0 = 0.564 cm and 1.2 cm for the parametric analysis. It can be seen from Figure 4.34 that the size of the slit influences the rate that the wetting front moves down into the soil. For example, after two years and at depth of 50 cm pressure head value for the smaller size slit was –75 cm, while value for the larger slit pressure head was –20 cm, which is an indication of higher moisture levels. At the other time intervals, the behavior is the same, but differences are less. For example, after ten years and at the depth of 70 cm, the pressure head was –8 cm for the smaller size slit, which indicates that, as time increases, the differences in the behavior becomes smaller. The size of the slit affects the leachate migration through the soil profile. Figure 4.34 shows that, for different time intervals, a higher amount of leakage occurs for larger slit sizes.

CHAPTER 5

CONCLUSIONS

In this study, a finite element computer program was developed (COMPBAR) to model flow through a composite barrier with a damaged geomembrane. The model may be used to solve the unsaturated transient flow condition as defined by Richards' equation. The model enables one to study the progress of a wetting front as it passes through the soil barrier component of a composite barrier. Using this model, one is able to estimate the following:

- Pressure head values at different locations in the soil profile as a function of time.

- Volumetric moisture content at different locations in the soil profile as a function of time.

- Values of hydraulic conductivity and flow rate throughout the depth of the soil profile as a function of time.

- Wetting front profiles as a function of time.

- The time required for the wetting front to reach the leachate detection layer and the amount of flow that reaches the leachate detection layer.

- The model can also be used to perform parametric studies of variables in the composite barrier system.

After analyzing different configurations of composite barriers with a damaged geomembrane, the following observations can be made.

- Pressure head values for the poor soil-geomembrane contact condition are less negative than pressure head values for the good soil-geomembrane contact condition at the same location and same time. This observations implies that a smooth soil surface with good soil-geomembrane contact results in longer life of the composite barrier system.

- At the same location, the volumetric moisture contents for the poor soil-geomembrane contact case are higher than those for the good soil-geomembrane contact, thus indicating that saturation levels are higher in the poor contact case than for the good contact case.

- The wetting front migrates slower through the soil in the good soil-geomembrane contact case than in the poor soil-geomembrane contact case.

- In poorly compacted soil, the wetting front moves rapidly while in the compacted clay barrier, there is gradual movement of the wetting front towards saturation.

- The higher the leachate head on top of the goemembrane, the faster the wetting front moves through the leachate detection layer. This observation implies that the pumping of excess leachate from the top of the geomembrane affects the leachate head which in turn affects the quantity of leachate through the geomembrane slits and into the soil profile.

- When the wetting radii of two different slits in the geomemnbrane overlap, the wetting front movement increases. Therefore, the more slits with close proximity, the more the flow rate of the wetting front movement is increased. This observation implies that careful monitoring of the condition of the geomembrane to detect slits, and all kinds of damage and the consequent wetting front movement, can help determine the effectiveness and longevity of the composite barrier system.

- The computer code, COMPBAR, has unique capabilities to model leachate flow through a composite barrier system in which multiple defects in the form of slits appear.

CHAPTER 6
RECOMMENDATIONS

The future research work should include the following.

- Expanding the model from its present two-dimensional capability to a three-dimensional capability so that more accurate results can be obtained.

- Determining the results of different configurations and sizes of damage in the geomembrane. A new model for leachate head must be developed because COMPBAR gives results with limited size and shape of geomembrane damage.

- It was assumed in this work that the head of the leachate was kept constant with time. A more precise approximation of wetting front movement would be obtained if a similar approximation for leachate head h (r) were obtained assuming transient conditions. To more accurately determine actual field conditions, a future finite element model should account for varying liquid levels over time, to simulate the evaporation or precipitation that naturally occurs on the composite barrier system.

- Developing small scale and large scale physical models, as well as actual composite barrier systems, to verify the results of the finite element program COMPBAR.

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COMPUTER PROGRAM

A.1 Overview

The computer program COMPBAR was developed to analyze the wetting front of the damaged composite barrier. The source code for the COMPBAR is written in Fortran 77 presented with all the subroutines needed to run the program.

A.2 Program Compbar

```
C*******************************************************************
C     THIS PROGRAM SIMULATES HIGHLY NONLINEAR TWO DIMENSIONAL
C     MOVEMENT OF LEACHATE WATER IN UNSATURATED COMPOSITE LINERS WITH
C     DAMAGED GEOMEMBRANE AND/OR TWO DIMENSIONAL TRANSIENT NONLINEAR
C     FOR SOIL PROFILE.
C     UPDATED: APRIL 15, 1994
C     THIS COMPUTER PROGRAM CONSISTS OF SUBROUTINES WHICH WERE ADAPTED
C     FROM PUBLISHED COMPUTER PROGRAMS. THE AUTHOR’S CONTRIBUTION IS
C     LIMITED TO THE ASSEMBLY OF THESE SUBROUTINES.
C*******************************************************************

      INCLUDE  'COMALL.IN'
      DIMENSION XX(MAX1),V(MAX2,3),NODETBL(13),XYZ(MAX1,3)
      INTEGER HDF,VLF
      LOGICAL LOOP,CONVRGE
      CHARACTER*20 INFILE,OUTFILE,PLTFILE
      CHARACTER*80 TITLE
      EQUIVALENCE (V1,V(1,1)),(V2,V(1,2)),(V3,V(1,3)),(X1,XYZ(1,1)),
     1              (X2,XYZ(1,2)),(X3,XYZ(1,3))
C       DATA MAXIT/20/,TOLRNCE/.5/
      DATA NODETBL/2,3,4,3,4,4,8,12,8,20,32,3,4/
C
      WRITE(*,*)'ENTER MAX # OF ITERATIONS, AND TOLERANCE:'
      READ(*,*)MAXIT,TOLRNCE
      SYMM = .TRUE.
      LOOP = .TRUE.
      LABEL1 = ' PRESSURE HEAD'
      LABEL2 = ' GROUNDWATER FLOW   '
      WRITE(*,10) ' ENTER THE NAME OF THE INPUT DATA FILE: '
 10   FORMAT(A)
      READ(*,20) INFILE
 20   FORMAT(A)
      WRITE(*,10) ' ENTER THE NAME OF THE OUTPUT FILE: '
      READ(*,20) OUTFILE
      OPEN(INF,FILE=INFILE,STATUS='UNKNOWN')
      OPEN(OUTF,FILE=OUTFILE,STATUS='NEW')
      READ(INF,20) TITLE
```

```
      WRITE(OUTF,20) TITLE
C
C   1.INPUT NODE NUMBERS AND COORDINATES
C
      READ(INF,*) DIM
C  IHOLE = 0 => WITHOUT HOLE; IHOLE >= 1 => WITH HOLE
      READ(INF,*)NOHOLE
      IF ( NOHOLE .GE. 1) THEN
      CALL HOLE
      WRITE(OUTF,*)'**************************************'
      WRITE(OUTF,*)'** QRATE = ',GQRATE,' m^3/sec ','**'

      WRITE(OUTF,*)'**************************************'
      ENDIF
      CALL NODES
C
C   2.INPUT ELEMENT NUMBERS AND ELEMENT NODE NUMBERS
C
      CALL ELEMENT
C
C   3.INPUT MATERIAL PROPERTIES FOR EACH ELEMENT
C
      CALL MATERL
C
C   4.INPUT BOUNDARY CONDITIONS FOR THIS PROBLEM
C
      IF (NOHOLE .GE. 1) CALL HBOUND
      CALL BOUND
C
C   5. INPUT INITIAL CONDITIONS
C
      WRITE(*,*)'NDN=',NDN
      CALL INITIAL
C
C   6. WRITE OUT CONTENTS OF ARRAYS
C
      CALL DUMP(LOOP,HDF,VLF)
      READ(INF,20)PLTFILE
      WRITE(*,10)PLTFILE
C
C    INITIALIZE COUNTERS
      IF (DIM .LE. 3) THEN
      IDIM = DIM
      ELSE
      IDIM = 2
      ENDIF
      IDT = 0
      IGT = 1
      IGTDT = 1
      T = 1.0
      ICOUNT = 0
C
C   7. FOR EACH TIME STEP
C
      DO 120 ISTEP = 1, MXSTEP
      T = T + DELTAT(IDT)
      DO 40 ITER = 1, MAXIT
C
       CALL PROPERTY
C
C         IF SIZE OF TIME STEP CHANGES REASSEMBLE GLOBAL MATRICES
```

```
        IF ((ITER .GT. 1) .OR. (ISTEP .EQ. 1)
     1       .OR. (ISTEP .GT. DTSTEP(IDT))) THEN
          IF (ITER .EQ. 1) IDT = IDT + 1
C           ASSEMBLE AND MODIFY THE GLOBAL SYSTEM OF EQUATIONS
          CALL ASMBKC
C           DECOMPOSE THE MODIFIED GLOBAL SYSTEM OF EQUATIONS
          CALL DECOMP(NDOF,SBW,SYMM,M)
         ENDIF
C         CALCULATE THE RIGHT HAND SIDE VECTOR FOR THIS TIME STEP
         CALL RHS
C         SOLVE THE SYSTEM OF EQUATIONS AND OUTPUT NODAL VALUES
C
         CALL SOLVE(NDOF,SBW,SYMM,M,B,XX)
         CONVRGE = .TRUE.

         II = NDOF
C
         DO 30 I = NUMNOD, 1, -1
          IF (ICH(I) .EQ. 0) THEN
C      WRITE(*,*)X(I),XX(II)
          IF (ABS((X(I) - XX(II))/XX(II)) .GT. TOLRNCE)
     1      CONVRGE = .FALSE.
           X(I) = XX(II)
           II = II - 1
          ENDIF
 30       CONTINUE
C
       DO E = 1, NUMELM
        NNOE = NODETBL(ELEMTYP(E))
        PRESSH(E) = 0.0
        DO J = 1, NNOE
         PRESSH(E) = PRESSH(E) + X(IN(E,J))/NNOE
        END DO
       END DO
C
        IF (CONVRGE) GOTO 50
 40     CONTINUE
        WRITE(*,20) ' *** EXCEEDS MAXIMUM NUMBER OF ITERATIONS ***'
 50     WRITE(*,*)'CONVERGENCE ACHIEVED',ISTEP
       WRITE(OUTF,60) LABEL1,LABEL1
 60     FORMAT(//70('*')//16X,'COMPUTED VALUES OF ',A/
     1      16X,39('-')//19X,'NODE NO.',10X,A/)
        DO 80 I = 1, NUMNOD
        IF (ICH(I) .EQ. 0) THEN
          WRITE(OUTF,70) I,X(I),' '
        ELSE
          WRITE(OUTF,70) I,X(I),'*'
        ENDIF
 70       FORMAT(19X,I5,12X,F15.4,A)
C        IF (HDF .NE. 0) WRITE(HDF,*) I,(XYZ(I,J),J=1,IDIM),X(I)
 80     CONTINUE
        WRITE(OUTF,90)
 90     FORMAT(/40X,'* = SPECIFIED')
        WRITE(OUTF,100) T
 100    FORMAT(/19X,'*** RESULTS FOR TIME =',F7.2,' ***')
        CALL VELOCITY
        WRITE(OUTF,100) T
        IF (VLF .NE. 0) THEN
        DO 110 I = 1, NUMELM
          WRITE(VLF,*) I,(V(I,J),J=1,IDIM)
 110      CONTINUE
```

```
      ENDIF
      WRITE(OUTF,201)
 201  FORMAT(T2,'ELEM #',T14,'K',T28,'C',T35,'FLOW RATE',T51
    1            ,'MOISTURE')
      DO IE = 1, NUMELM
     WRITE(OUTF,202)IE,PROP(IE,1),PROP(IE,DIM+1),QRATE(IE),
    1                  XMOIST1(IE)
      END DO
 202 FORMAT(I4,2X,1P3E13.4,2X,1P3E13.4,2X,1P3E13.4
    1                  ,(2X,1P3F13.2))
     IF (HDF .NE. 0) THEN
       ICOUNT = ICOUNT + 1
       CALL PLOT(0,IDIM,ICOUNT,PLTFILE,XYZ,T)
     ENDIF
 120 CONTINUE
     CALL PLOT(1,IDIM,ICOUNT,PLTFILE,XYZ,T)
     END




     SUBROUTINE PROPERTY
C*************************************************************************
C
C    PROPERTY SUBROUTINE TO COMPUTES THE VALUES OF THE HYDRAULIC CONDUCTIVITY
C    (K)AND SPECIFIC MOISTURE CAPACITY (C) FOR EVERY ELEMENT IN THE MESH
C    EMPERICALLY FOR EVERY PICARD ITERATION OR TIME ITERATION.  WHEN GO TO
C    NEXT STEP OBTAIN NEW VALUES OF K AND C VALUES APPROXIMATELY AGAIN.
C    TO BE ABLE TO OBTAIN THE VERSION OF K INSIDE PROPERTY SUBROUTINE,
C    INITIAL PRESSURE HEAD IN THE ELELEMENT SHOULD BE KNOWN.
C
C    DEFINITION OF THE PARAMETERS:
C
C    ALPHA        =     SCALING LENGTH  (cm-1)
C    NN           =     POROSITY
C    MM           =     1- 1/NN
C    PRESS(E)     =     PRESSURE HEAD OF THE ELEMENT
C    MODTYP       =     TYPE OF MODEL
C                 0:    SATURATED SOIL
C                 1:    BURDINE'S MODEL
C                 2:    MUALEM'S MODEL
C                 3:    GENUNCHTEN'S MODEL
C                 4:    GARDNER'S MODEL
C                 5,6:  SATURATED SOIL
C    THETAS       =     SATURATED WATER CONTENT (cm3/cm3)
C    THETAR       =     RESIDUAL WATER CONTENT (cm3/cm3)
C    MATSET(E)    =     SET OF MATERIAL OF SOIL
C    PROPI        =     PROPERTY INITIAL
C    ELMTHIK      =     THICKNES OF THE ELEMENT
C    ELMWIDT      =     WIDTH OF THE ELEMENT
C    QRATE        =     OVERALL FLOW RATE
C    ABS          =     ABSOLUTE VALUE
C
C*************************************************************************
C
C
     SUBROUTINE PROPERTY
C
     INCLUDE  'COMALL.IN'
C
```

```
      REAL MM,NN,ALPHA
C
      DO E = 1, NUMELM
       POROSITY = PROPI(MATSET(E),DIM+7)
       ALPHA = PROPI(MATSET(E),2+DIM)
       NN = PROPI(MATSET(E),3+DIM)
       MM = PROPI(MATSET(E),4+DIM)
       IF(MODTYP.EQ.1)THEN
C  BURDINE'S MODEL
      IF (PRESSH(E) .GE. 0.0)THEN
       RATIO = 1.0
       PROP(E,DIM+1) = 0.0
      ELSE
         RATIO = (1.0-(ALPHA*(-PRESSH(E)))**(NN-2.)*
     1         (1.0+(ALPHA*(-PRESSH(E)))**NN)**(-MM))/
     2         ((1.0+(ALPHA*(-PRESSH(E)))**NN)**(2.*MM))
         PROP(E,DIM+1) = ALPHA*MM*NN*(ALPHA*(-PRESSH(E)))**(NN-1.)/
     1          (1.0+(ALPHA*(-PRESSH(E)))**NN)**(MM+1.)
      ENDIF
       ELSEIF(MODTYP.EQ.2)THEN
C  MUALEM'S MODEL
      IF (PRESSH(E) .GE. 0.0) THEN
       RATIO = 1.0
       PROP(E,DIM+1) = 0.0
      ELSE
         RATIO = (1.0-(ALPHA*(-PRESSH(E)))**(NN-1.0)
     1          *(1.0+(ALPHA*(-PRESSH(E)))**NN)**(-MM))**2.
     2          /(1.0+(ALPHA*(-PRESSH(E)))**NN)**(MM/2.)
         PROP(E,DIM+1) = ALPHA*MM*NN*(ALPHA*(-PRESSH(E)))**(NN-1.)/
     1          (1.0+(ALPHA*(-PRESSH(E)))**NN)**(MM+1.)
      ENDIF
         ELSEIF(MODTYP.EQ.3)THEN
C  GENUCHTEN'S MODEL
         THETAS = PROPI(MATSET(E),DIM+5)
         THETAR = PROPI(MATSET(E),DIM+6)
      IF (PRESSH(E) .GE. 0.0) THEN
       RATIO = 1.0
       PROP(E,DIM+1) = 0.0
      ELSE
         RATIO = (1.0-(ALPHA*(-PRESSH(E)))**(NN-1)*
     1          (1.0+(ALPHA*(-PRESSH(E)))**NN)**(-MM))**2/
     2          (1.0+(ALPHA*(-PRESSH(E)))**NN)**(MM/2.)
         PROP(E,DIM+1) = ALPHA*MM*NN*(ALPHA*(-PRESSH(E)))**(NN-1)*
     1          (THETAS-THETAR)/(1.0+(ALPHA*(-PRESSH(E)))**NN)**(MM+1)
      ENDIF
         ELSEIF(MODTYP.EQ.4)THEN
C  GARDNER'S MODEL (USED BY ISTOK & SOILINER'S MODEL)
      IF (PRESSH(E) .GE. 0.0)THEN
       RATIO = 1.0
       PROP(E,DIM+1) = 0.0
      ELSE
         RATIO = EXP(-ALPHA*PRESSH(E))
         PROP(E,DIM+1) = NN/(LOG(10.0)*PRESSH(E))
      ENDIF
       ELSE
        RATIO = 1.0
        PROP(E,DIM+1) = PROPI(MATSET(E),DIM+1)
       ENDIF
      IF(RATIO.GE.1.0) RATIO = 1.0
        DO I = 1,DIM
         PROP(E,I) = PROPI(MATSET(E),I) * RATIO
```

```
        END DO
C      COMPUTE THE FLOW RATE AND THE VOLUMETRIC MOISTURE CONTENT
C      IN EVERY ELEMENT APPROXIMATELY
C
       ELMTHIK = 0.5 * (ABS(X2(IN(E,4)) - X2(IN(E,1))) +
     1                 ABS(X2(IN(E,3))-  X2(IN(E,2))))
       ELMWIDT = 0.5 * (ABS(X1(IN(E,3)) - X1(IN(E,4))) +
     1                 ABS(X1(IN(E,2)) - X1(IN(E,1))))
       QRATE(E) = PROP(E,1)*PRESSH(E)*ELMWIDT/ELMTHIK
        XMOIST1(E) = POROSITY*RATIO
      IF (PRESSH(E) .GE. 0.0) THEN
       XMOIST2(E) = POROSITY
      ELSE
         XMOIST2(E) = POROSITY*(1.0/(1.0+(ALPHA*(-PRESSH(E)))**NN))**MM
      ENDIF
      END DO
      RETURN
      END




      SUBROUTINE PLOT
C******************************************************************
C
C     SUBROUTINE PLOT CREATES AND SAVES PRN FILES FOR PLOTTING.
C
C     DEFINITION OF THE PARAMETERS:
C
C     PLTFILE     =     FILE OF PLOT
C     IDIM        =     DIMENSION
C     KFILE       =     FILE OF HYDRAULIC CONDUCTIVITY (K)
C     QFILE       =     FILE OF FLOW RATE (Q)
C     MFILE       =     FILE OF SPECIFIC MOISTURE CAPACITY (C)
C     XFILE       =     FILE OF VOLUMMETRIC MOISTURE CONTENT (θ)
C     MXSTEP      =     MAXIMUM STEP
C     NUMELM      =     NUMBER OF ELEMENTS
C
C******************************************************************
      SUBROUTINE PLOT(IPLTCOD,IDIM,ICOUNT,PLTFILE,XYZ,TP)
      INCLUDE 'COMALL.IN'
      INTEGER ICOUNT,IE,I,IPLTCOD,J,IDIM
      REAL TP,PX(0:MAX1,50),PMST(0:MAX2,50),PQ(0:MAX2,50),
     +      PK(0:MAX2,50),PEXYZ(0:MAX2,3),XYZ(MAX1,3)
      CHARACTER *20 PLTFILE,KFILE,QFILE,MFILE,XFILE
C
      IF(IPLTCOD .EQ. 0) THEN
       PX(0,ICOUNT) = TP
       PMST(0,ICOUNT) = TP
       PQ(0,ICOUNT) = TP
       PK(0,ICOUNT) = TP
       DO I = 1, NUMNOD
        PX(I,ICOUNT) = X(I)
       END DO
       DO IE = 1, NUMELM
        PMST(IE,ICOUNT) = XMOIST1(IE)
        PQ(IE,ICOUNT) = QRATE(IE)
        PK(IE,ICOUNT) = PROP(IE,1)
       END DO
      ELSEIF(IPLTCOD .EQ. 1) THEN
       DO IE = 1, NUMELM
```

```
          DO J = 1, IDIM
           PEXYZ(IE,J) = (XYZ(IN(IE,1),J) + XYZ(IN(IE,3+J-1),J))/2.0
          END DO
         END DO
         KFILE = PLTFILE(1:6)//'-1'//'.PRN'
         QFILE = PLTFILE(1:6)//'-2'//'.PRN'
         MFILE = PLTFILE(1:6)//'-3'//'.PRN'
         XFILE = PLTFILE(1:6)//'-4'//'.PRN'
         OPEN(UNIT=1,FILE=KFILE,STATUS='UNKNOWN')
          DO IE = 0, NUMELM
           WRITE(1,*)IE,(PEXYZ(IE,J),J=1,IDIM),(PK(IE,J),J=1,MXSTEP)
          END DO
         CLOSE(1)
         OPEN(UNIT=1,FILE=QFILE,STATUS='UNKNOWN')
          DO IE = 0, NUMELM
           WRITE(1,*)IE,(PEXYZ(IE,J),J=1,IDIM),(PQ(IE,J),J=1,MXSTEP)
          END DO
         CLOSE(1)
         OPEN(UNIT=1,FILE=MFILE,STATUS='UNKNOWN')
          DO IE = 0, NUMELM
           WRITE(1,*)IE,(PEXYZ(IE,J),J=1,IDIM),(PMST(IE,J),J=1,MXSTEP)
          END DO
         CLOSE(1)
         OPEN(UNIT=1,FILE=XFILE,STATUS='UNKNOWN')
          DO I = 0, NUMNOD
           WRITE(1,*)I,(XYZ(I,J),J=1,IDIM),(PX(I,J),J=1,MXSTEP)
          END DO
         CLOSE(1)
        END IF
        RETURN
        END
```

```
C**********************************************************************
C                       FILE "COMALL"
C
C     THIS FILE DIMENSIONS ARRAYS FOR  THOSE VARIABLES SHARED
C     BY THE SUBROUTINES. WHEN THE SUBROUTINES ARE
C     COMPILED, THE STATEMENT
C                    $INCLUDE: 'COMALL'
C     THAT APPEARS IN EACH SUBROUTINE WILL DIRECT THE COMPILER TO
C     PROCEED AS THOUGH THE SPECIFIED FILE (COMALL) WERE INSERTED
C     AT THE POINT OF THE $INCLUDE.
C
C     DEFINITION OF PARAMETERS:
C
C     INF  = UNIT SPECIFICATION FOR INPUT DATA FILE
C     OUTF = UNIT SPECIFICATION FOR OUTPUT FILE
C     MAX1 = MAXIMUM NUMBER OF NODES
C     MAX2 = MAXIMUM NUMBER OF ELEMENTS
C     MAX3 = MAXIMUM NUMBER OF NODES PER ELEMENT
C     MAX4 = MAXIMUM NUMBER OF MATERIAL SETS
C     MAX5 = MAXIMUM NUMBER OF MATERIAL PROPERTIES PER MATERIAL SET
C     MAX6 = MAXIMUM VALUE OF SEMI-BANDWIDTH
C     MAX7 = MAXIMUM NUMBER OF DIFFERENT TIME STEP INCREMENTS
C     MAX8 = MAXIMUM SIZE OF MODIFIED GLOBAL CONDUCTANCE MATRIX IN
C            VECTOR STORAGE
C
```

```
C*********************************************************************
        REAL M,B1
        INTEGER OUTF,DIM,ELEMTYP,SBW,E,DTSTEP
        LOGICAL SYMM
        CHARACTER*25 LABEL1,LABEL2
        PARAMETER (INF=5,OUTF=6)
        PARAMETER (MAX1=1000,MAX2=1000,MAX3=32,MAX4=200,
     1             MAX5=32,MAX6=1000,MAX7=20,MAX8=90000)
        COMMON /COM1/ DIM,NUMNOD,NUMELM,NUMMAT,NUMPROP,
     1                NDN,NNN,NDOF,SBW,ICH(MAX1),LCH(MAX1),
     2                X(MAX1),FLUX(MAX1),B(MAX1),X1(MAX1),
     3                X2(MAX1),X3(MAX1),SYMM,LABEL1,LABEL2
        COMMON /COM2/ IN(MAX2,MAX3),ELEMTYP(MAX2),V1(MAX2),
     1                V2(MAX2),V3(MAX2)
        COMMON /COM3/ MATSET(MAX2),PROP(MAX2,MAX5),
     1                PRESSH(MAX2),PROPI(MAX2,MAX5)
        COMMON /TFUNC/ FC(MAX1),DTSTEP(MAX7),DELTAT(MAX7),
     1                 TIME(MAX7),GT(MAX7),OMEGA,OMOMEGA,
     2                 MXSTEP,T,IDT,IGT,IGTDT
        COMMON /GHOLE/ NOHOLE,HCOORD(MAX1),HR0I(MAX1),
     1                 RWETI(MAX1),HHWI(MAX1),MODTYP,XMOIST1(MAX2),
     2                   XMOIST2(MAX2),POROSITY,GQRATE
        COMMON /GLOBAL/ M(MAX8),QRATE(MAX2)
        COMMON /GLOB/ B1(MAX8)




      SUBROUTINE HOLE
C************************************************************************

C     SUBROUTINE HOLE CREATES THE PRESSURE HEAD (H(X)) FOR EVERY HOLE
C     THE PARABOLIC VARIATION OF THE PRESSURE HEAD DUE TO EVERY HOLE.
C     USING THE GIROUD EQUATION,  IT GIVES AS INPUT AND READS RADIUS OF
C     HOLE (R0), PRESSURE HEAD OF THE LEACHATE (Hw),  AND HYDRAULIC
C     CONDUCTIVITY (K) OF THE CLAY LINER FOR EVERY HOLE.
C     OUTPUT ARE WETTED RADIUSES,  PRESSURE HEAD (Hw) VALUES AND PARABOLIC
C     VARIATION OF PRESSURE HEAD (Hw) FOR EVERY HOLE.
C
C      DEFINITION OF THE PARAMETERS:
C
C     HUNITS      =     UNITS OF MEASURE OF THE PROBLEM
C     NOHOLE      =     NUMBER OF HOLES
C     GQRATE      =     OVERALL FLOW RATE
C     HCOORD      =     COORDINATES OF THE HOLES
C     IHTYPE      =     TYPE OF HOLES
C                       0:    CIRCULAR
C                       1:    SQUARE
C     IHCONT      =     TYPE OF CONTACT BETWEEN GEOMEMBRANE AND CLAY LINER
C     HRO         =     RADIUS OF THE HOLE
C     HHW         =     HEIGHT OF LEACHATE
C     HKS         =     HYDRAULIC CONDUCTIVITY OF THE CLAY LINER
C     AHOLE       =     AREA OF THE HOLE
C     WETF        =     WETTING FRONT
C     RWET        =     RADIUS OF THE WETTED AREA
C
C************************************************************************
      INCLUDE 'COMALL.IN'
      GQRATE = 0.0
      READ(INF,*)HUNITS
      DO I = 1, NOHOLE
C  ENTER THE LOCATION OF THE HOLE ALONG THE X-AXIS, USE GLOBAL UNITS
       READ(INF,*)HCOORD(I)
```

```
C
C  ENTER TYPE OF HOLE (CIRCULAR = 0, SQUARE = 1); GOOD = 0, POOR = 1
        READ(INF,*)IHTYPE, IHCONT
C  ENTER THE FOLLOWING PARAMETERS IN m & sec. UNITS
C
        READ(INF,*)HR0, HHW, HKS
       write(*,*)I,hr0,hhw,hks
        AHOLE = 3.14 * HR0**2
        IF (IHTYPE .EQ. 1) AHOLE = 4.0 * HR0**2
        WETF = 0.26
        IF ( IHCONT .EQ. 1) WETF = 0.61
        QF = 0.21
        IF ( IHCONT .EQ. 1) QF = 1.15
C  RADIUS OF WETTED AREA (METERS)
        RWET = WETF * AHOLE**0.05 * HHW**0.45 * HKS**(-0.13)
       QRATEI = QF * AHOLE**0.1 * HHW**0.9 * HKS**0.74
C  ENTER THE UNITS OF MEASURE OF THE PROBLEM
        IF ( HUNITS .EQ. 1) THEN
         RWET = RWET * 100.0
         HR0 = HR0 * 100.0
         HHW = HHW * 100.0
        ELSEIF ( HUNITS .EQ. 2) THEN
         RWET = RWET * 3.281
         HR0 = HR0 * 3.281
         HHW = HHW * 3.281
        ELSEIF ( HUNITS .EQ. 3) THEN
         RWET = RWET * 39.37
         HR0 = HR0 * 39.37
         HHW = HHW * 39.37
        ENDIF
C
        RWETI(I) = RWET
        HR0I(I) = HR0
        HHWI(I) = HHW
       GQRATE = GQRATE + QRATEI
       END DO
C
       RETURN
       END




SUBROUTINE NODES
C*********************************************************************
C
C         TO INPUT NODE NUMBERS AND COORDINATES
C
C   INPUT:
C         NODE NUMBERS AND COORDINATES ARE READ FROM THE USER-
C         SUPPLIED FILE ASSIGNED TO UNIT "INF"
C
C   OUTPUT:
```

```
C          NODE NUMBERS AND COORDINATES ARE WRITTEN TO THE USER-
C          DEFINED FILE ASSIGNED TO UNIT "OUTF"
C
C   DEFINITIONS OF VARIABLES:
C             DIM = COORDINATE SYSTEM TYPE
C             INC = NODE NUMBER INCREMENT
C          NUMNOD = NUMBER OF NODES READ
C           X1(I) = X COORDINATE FOR NODE I IF DIM = 1, 2
C           X2(I) = IS NOT USED IF DIM = 1
C                 = Y COORDINATE FOR NODE I IF DIM = 2
C
C   USAGE:
C          NODE NUMBERS AND COORDINATES ARE READ, ONE NODE
C          PER LINE, BEGINNING WITH NODE 1.  NODE NUMBERS
C          FOR "MISSING" NODES ARE GENERATED BY THE
C          SUBROUTINE BY ADDING THE NODE NUMBER INCREMENT
C          TO THE NODE NUMBER FOR THE PRECEEDING NODE.
C          COORDINATES FOR "MISSING" NODES ARE COMPUTED BY
C          THE SUBROUTINE USING LINEAR INTERPOLATION.
C
C        SUBROUTINES CALLED:
C          NONE
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
        DIMENSION XYZ(MAX1,3)
        EQUIVALENCE (X1,XYZ(1,1)),(X2,XYZ(1,2)),(X3,XYZ(1,3))
        INTEGER CNODE,OLDNOD
C
        NUMNOD = 0
        IDIM = DIM
        IF (DIM .EQ. 4) IDIM = 2
        OLDNOD = MAX1
C       READ FROM INPUT FILE: NODE NUMBER, NODE NUMBER INCREMENT,
C       AND NODAL COORDINATES
 10     READ(INF,*) CNODE,INC,(XYZ(CNODE,I),I=1,IDIM)
        IF (CNODE .EQ. -1) GOTO 40
        IF (CNODE .GT. NUMNOD) NUMNOD = CNODE
C       GENERATE NODE NUMBERS AND COORDINATES FOR "MISSING" NODES
        NGENP1 = (CNODE - OLDNOD) / INC
        IF (NGENP1 .GT. 0) THEN
          DO 30 I = 1, IDIM
            XYZINC = (XYZ(CNODE,I) - XYZ(OLDNOD,I)) / FLOAT(NGENP1)
            DO 20 J = OLDNOD + INC, CNODE - INC, INC
              XYZ(J,I) = XYZ(J-INC,I) + XYZINC
 20         CONTINUE
 30       CONTINUE
        ENDIF
        OLDNOD = CNODE
        GOTO 10
C       WRITE NODE NUMBERS AND NODAL COORDINATES TO OUTPUT FILE
 40     IF (NUMNOD .GT. 0) THEN
          IF (DIM .EQ. 1) THEN
            WRITE(OUTF,50)
 50         FORMAT(3X,'NODE',10X,'NODAL COORDINATES'/
     1             2X,'NUMBER',18X,'X'/
     2             2X,6('-'),8X,20('-'))
          ELSEIF (DIM .EQ. 2) THEN
            WRITE(OUTF,60)
 60         FORMAT(3X,'NODE',21X,'NODAL COORDINATES'/
     1             2X,'NUMBER',18X,'X',20X,'Y'/
```

```
    2             2X,6('-'),8X,20('-'),1X,20('-'))
          ELSEIF (DIM .EQ. 3) THEN
            WRITE(OUTF,70)
 70         FORMAT(3X,'NODE',31X,'NODAL COORDINATES'/
    1             2X,'NUMBER',18X,'X',20X,'Y',20X,'Z'/
    2             2X,6('-'),8X,20('-'),1X,20('-'),1X,20('-'))
          ELSEIF (DIM .EQ. 4) THEN
            WRITE(OUTF,80)
 80         FORMAT(3X,'NODE',21X,'NODAL COORDINATES'/
    1             2X,'NUMBER',18X,'R',20X,'Z'/
    2             2X,6('-'),8X,20('-'),1X,20('-'))
          ENDIF
          DO 100 I = 1, NUMNOD
            WRITE(OUTF,90) I,(XYZ(I,J),J=1,IDIM)
 90         FORMAT(I6,10X,3(F15.4,6X))
 100      CONTINUE
        ELSE
          WRITE(OUTF,110)
 110      FORMAT(' NO NODAL POINT DATA READ.')
        ENDIF
        RETURN
        END
```

```
      SUBROUTINE ELEMENT
C*********************************************************************
C
C
C         TO INPUT ELEMENT NUMBERS, TYPES,  NODE NUMBERS
C
C  INPUT:
C         ELEMENT NUMBERS, TYPES, AND NODE NUMBERS ARE READ
C         FROM THE USER-SUPPLIED FILE ASSIGNED TO UNIT "INF"
C
C  OUTPUT:
C         ELEMENT NUMBERS, TYPES, AND NODE NUMBERS ARE WRITTEN
C         TO THE USER-DEFINED FILE ASSIGNED TO UNIT "OUTF"
C
C  DEFINITIONS OF VARIABLES:
C         ELEMTYP(I) = ELEMENT TYPE FOR ELEMENT I
C            IN(I,J) = NODE NUMBER J FOR ELEMENT I
C                INC = NODE NUMBER INCREMENT
C         NODETBL(I) = NUMBER OF NODES IN ELEMENT TYPE I
C             NUMELM = NUMBER OF ELEMENTS IN MESH
C
C  USAGE:
C         ELEMENT DATA (ELEMENT NUMBER, TYPE, AND NODE NUMBERS)
C         ARE READ SEQUENTIALLY, SET OF ELEMENT DATA PER LINE.
C         ELEMENT NUMBERS, TYPES, AND NODE NUMBERS FOR "MISSING"
C         ELEMENTS ARE GENERATED BY THE SUBROUTINE.
C
C       SUBROUTINES CALLED:
C         NONE
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
        INTEGER OLDELM,ELM,TYPE
        DIMENSION NODETBL(13)
        DATA NODETBL/2,3,4,3,4,4,8,12,8,20,32,3,4/
```

```
C
        MAXNOD=0
        OLDELM=MAX2
        NUMELM=0
C       READ FROM INPUT FILE: ELEMENT NUMBER, ELEMENT TYPE,
C       AND ELEMENT NODE NUMBERS
 10     READ(INF,*) ELM,TYPE,INC,(IN(ELM,I),I=1,NODETBL(ABS(TYPE)))
        IF (ELM .EQ. -1) GOTO 40
        ELEMTYP(ELM) = TYPE
        IF (ELM .GT. NUMELM) NUMELM = ELM
        IF (NODETBL(TYPE) .GT. MAXNOD) MAXNOD = NODETBL(TYPE)
C       GENERATE THE MISSING ELEMENTS
        IF (ELM .GT. OLDELM+1) THEN
          DO 30 I = OLDELM + 1, ELM -1
            IM1 = I - 1
            DO 20 J = 1, NODETBL(TYPE)
                IN(I,J) = IN(IM1,J) + INC
 20         CONTINUE
            ELEMTYP(I) = TYPE
 30       CONTINUE
        ENDIF
        OLDELM = ELM
        GOTO 10
C       WRITE ELEMENT NUMBERS AND ELEMENT NODE NUMBERS TO OUTPUT FILE
 40     IF (NUMELM .GT. 0) THEN
          IF (MAXNOD .EQ. 2) THEN
            WRITE(OUTF,50) (' ',I=1,2)
          ELSEIF (MAXNOD .EQ. 3) THEN
            WRITE(OUTF,50) ('     ',I=1,2)
          ELSEIF (MAXNOD .EQ. 4) THEN
            WRITE(OUTF,50) ('        ',I=1,2)
          ELSEIF (MAXNOD .GT. 4) THEN
            WRITE(OUTF,50) ('                     ',I=1,2)
          ENDIF
 50       FORMAT(/,2(2X,'ELEMENT',4X)/4X,'NO.',10X,'TYPE',6X,A,
     1           'NODE NUMBERS'/2(2X,'-------',4X),1X,A,'------------')
          DO 70 I = 1, NUMELM
            WRITE(OUTF,60) I,ELEMTYP(I),
     1                   (IN(I,J),J=1,NODETBL(ELEMTYP(I)))
 60         FORMAT(I7,I13,6X,8I6:4(/26X,8I6))
 70       CONTINUE
        ELSE
          WRITE(OUTF,80)
 80       FORMAT(' NO ELEMENT DATA READ.')
        ENDIF
        RETURN
        END




      SUBROUTINE MATERL
C*********************************************************************
C
C         TO INPUT ELEMENT MATERIAL SET NUMBERS AND MATERIAL
C         PROPERTIES FOR EACH MATERIAL SET
C
C  INPUT:
C         ELEMENT MATERIAL SET NUMBERS AND MATERIAL PROPERTIES
C         FOR EACH MATERIAL SET ARE READ FROM THE USER-SUPPLIED
C         FILE ASSIGNED TO UNIT "INF"
C
```

```
C  OUTPUT:
C         ELEMENT MATERIAL SET NUMBERS AND MATERIAL PROPERTIES
C         FOR EACH MATERIAL SET ARE WRITTEN TO THE USER-DEFINED
C         FILE ASSIGNED TO UNIT "OUTF"
C
C  DEFINITIONS OF VARIABLES:
C          MATSET(I) = MATERIAL SET NUMBER FOR ELEMENT I
C             NUMMAT = NUMBER OF MATERIAL SETS
C            NUMPROP = NUMBER OF MATERIAL PROPERTIES IN EACH
C                      MATERIAL SET
C          PROP(I,J) = MATERIAL PROPERTY J FOR MATERIAL SET I
C
C  USAGE:
C         ELEMENT MATERIAL SET NUMBERS ARE READ IN SEQUENTIALLY,
C         ONE ELEMENT NUMBER AND MATERIAL SET NUMBER PER LINE.
C         MATERIAL SET NUMBERS FOR "MISSING" ELEMENTS ARE
C         GENERATED BY THE SUBROUTINE BY ASSIGNING THE MATERIAL
C         SET NUMBER OF THE PRECEEDING ELEMENT TO EACH "MISSING"
C         ELEMENT.
C
C       SUBROUTINES CALLED:
C         NONE
C
C*********************************************************************
      INCLUDE  'COMALL.IN'
      INTEGER OLDELM,ELM,SETNUM
C
      OLDELM = MAX4
      NUMMAT = 0
C       READ FROM INPUT FILE: ELEMENT NUMBER, AND MATERIAL SET NUMBER
 10     READ(INF,*) ELM,MATSET(ELM)
      IF (ELM .EQ. -1) GOTO 30
C       DETERMINE THE NUMBER OF MATERIAL SETS
      IF (MATSET(ELM) .GT. NUMMAT) NUMMAT = MATSET(ELM)
C       GENERATE THE MATERIAL SET NUMBER FOR EACH "MISSING" ELEMENT
      IF (ELM .GT. OLDELM + 1) THEN
        DO 20 I = OLDELM + 1, ELM - 1
          MATSET(I) = MATSET(I-1)
 20       CONTINUE
      END IF
      OLDELM = ELM
      GOTO 10
C       WRITE THE MATERIAL SET NUMBER FOR EACH ELEMENT TO OUTPUT FILE
 30     IF (NUMELM .GT. 0) THEN
        WRITE(OUTF,40)
 40       FORMAT(//2X,'ELEMENT'/4X,'NO.',9X,'MATERIAL SET NUMBER'/
     1           2X,'-------',7X,'--------------------')
        DO 60 I = 1, NUMELM
          WRITE(OUTF,50) I,MATSET(I)
 50         FORMAT(I6,I20)
 60       CONTINUE
C         READ FROM INPUT FILE: THE NUMBER OF PROPERTIES IN EACH MATERIAL SET
        READ(INF,*) NUMPROP
        IF (NUMPROP .EQ. 1) THEN
          WRITE(OUTF,70) (' ',I=1,2)
        ELSEIF (NUMPROP .EQ. 2) THEN
          WRITE(OUTF,70) ('         ',I=1,2)
        ELSEIF (NUMPROP .EQ. 3) THEN
          WRITE(OUTF,70) ('                 ',I=1,2)
        ELSEIF (NUMPROP .GE. 4) THEN
```

```
          WRITE(OUTF,70) ('                       ',I=1,2)
        ENDIF
 70       FORMAT(//2X,'MATERIAL'/3X,'SET NO.',3X,A,
     1           'MATERIAL  PROPERTIES'/2X,'--------',3X,A,
     2           '--------------------')
C         WRITE MATERIAL PROPERTIES INFORMATION TO OUTPUT FILE
        DO 90 I = 1, NUMMAT
           READ(INF,*) SETNUM,MODTYP,(PROPI(SETNUM,J),J=1,NUMPROP)
C
C            READ(INF,*) SETNUM,(PROP(SETNUM,J),J=1,NUMPROP)
C            WRITE(OUTF,80) SETNUM,(PROP(SETNUM,J),J=1,NUMPROP)
 80         FORMAT(I7,7X,8(1P4E15.6/14X))
 90       CONTINUE
        READ(INF,*)E,PRESSH(E)
        DO E = 2, NUMELM
          PRESSH(E) = PRESSH(E-1)
        END DO
C
      ELSE
        WRITE(OUTF,100)
 100      FORMAT(' NO ELEMENT MATERIAL PROPERTY DATA READ.')
      ENDIF
      RETURN
      END




      SUBROUTINE HBOUND
C***********************************************************************
C     SUBROUTINE HBOUND COMPUTES THE PRESSURE HEAD (Hw) DUE TO HOLES AS
C     THE CONTRIBUTION OF THE ALL THE NODES IF THE HOLES ARE OVERLAPPED.
C     INPUT:THE FIRST NODE ON TOP OF THE MESH.
C           THE LAST NODE ON TOP OF THE MESH.
C           INCREMENT FOR THE NODES FOR SPECIFIED PRESSURE HEAD.
C
C           DEFINITION OF THE PARAMETERS:
C
C           NUMNOD      =     NUMBER OF NODES
C           HN1         =     HOLE NUMBER 1
C           HN2         =     HOLE NUMBER 2
C           HINC        =     HOLE INCREMENT
C           HCOORD      =     HCOORDINATES
C           ABS         =     ABSOLUTE VALUE
C           HVALUE      =     VALUE OF THE HOLE
C           HHI         =     HOLE INCREMENT
C
C***********************************************************************
      INCLUDE 'COMALL.IN'
C
      NDN = 0
      DO K = 1, NUMNOD
       ICH(K) = 0
      END DO
      READ(INF,*)HN1,HN2,HINC
      DO I = HN1, HN2, HINC
       HHI = 0.0
       DO J = 1, NOHOLE
        HRI = ABS(HCOORD(J) - X1(I))
        IF (HRI .EQ. 0.0) THEN
         HVALUE = HHWI(J)
        ELSE
         HVALUE = HHWI(J) * LOG(RWETI(J)/HRI)/LOG(RWETI(J)/HR0I(J))
```

```
          ENDIF
          IF (HVALUE .LE. 0.0) HVALUE = 0.0
          HHI = HHI + HVALUE
         END DO
         X(I) = -HHI
         ICH(I) = 1
         NDN = NDN + 1
        END DO
        RETURN
        END
```

```
      SUBROUTINE BOUND
C**********************************************************************
C
C     TO INPUT SPECIFIED VALUES OF THE HYDRAULIC HEAD AND THE PRESSURE
C     HEAD FOR SELECTED NODES.
C
C  INPUT:
C     SPECIFIED VALUES OF THE HYDRAULIC HEAD AND PRESSURE HEAD
C     ARE READ FROM THE USER-SUPPLIED FILE ASSIGNED TO UNIT "INF".
C
C  OUTPUT:
C          SPECIFIED VALUES OF ARE THE HYDRAULIC HEAD AND PRESSURE HEAD
C          WRITTEN TO THE USER-DEFINED FILE ASSIGNED TO UNIT "OUTF".
C
C  DEFINITIONS OF VARIABLES:
C          FLUX(I) = SPECIFIED VALUE OF GROUNDWATER FLOW AT NODE I
C           ICH(I) = 1 IF THE VALUE OF THE FIELD VARIABLE IS
C                    SPECIFIED FOR NODE I,
C                  = 0 OTHERWISE
C           LCH(I) = ICH(I) + ICH(I-1) + ICH(I-2) + ...
C                    THE ARRAYS ICH AND LCH ARE USED TO MODIFY
C                    GLOBAL SYSTEM OF EQUATIONS IN SUBROUTINES
C                    ASMBK, ASMBKC, AND ASMBAD
C           LABEL1 = CHARACTER VARIABLE USED TO LABEL COLUMN
C                    HEADINGS FOR SPECIFIED VALUES OF THE FIELD
C                    VARIABLE ON FILE ASSIGNED TO UNIT OUTF.
C                    LABEL1 = "HYDRAULIC HEAD", "PRESSURE HEAD"
C           LABEL2 = CHARACTER VARIABLE USED TO LABEL COLUMN HEADINGS
C                    FOR SPECIFIED VALUES OF GROUNDWATER FLOW ON FILE
C                    ASSIGNED TO UNIT OUTF.
C                    LABEL2 = "GROUNDWATER FLOW" OR "SOLUTE FLUX"
C              NDN = NUMBER OF NODES WITH SPECIFIED VALUES OF THE
C                    FIELD VARIABLE (NAMED FOR NUMBER OF DIRICHLET
C                    NODES)
C             NDOF = NUMBER OF NODES WHERE THE VALUE OF THE FIELD
C                    VARIABLE IS UNKNOWN (NAMED FOR NUMBER OF DEGREES
C                    OF FREEDOM)
C              NNN = NUMBER OF NODES WITH SPECIFIED VALUES OF
C                    GROUNDWATER FLOW.
C             X(I) = SPECIFIED VALUE OF THE FIELD VARIABLE
C                    (HYDRAULIC HEAD, PRESSURE HEAD AT NODE I
C
C   USAGE:
C          SPECIFIED VALUES OF THE FIELD VARIABLE ARE READ FIRST, ONE
C          NODE NUMBER AND THE SPECIFIED VALUE OF THE FIELD VARIABLE
C          AT THAT NODE PER LINE.  THE NODE NUMBERS CAN BE LISTED IN
C          ANY ORDER ON THE INPUT FILE.  THE VALUE OF THE FIELD
```

```
C          VARIABLE MUST BE SPECIFIED FOR AT LEAST ONE NODE IN THE
C          MESH.
C
C       SUBROUTINES CALLED:
C         NONE
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
C
C       INITIALIZATION
        DO 10 I = 1, NUMNOD
C          ICH(I) = 0
          FLUX(I) = 0.
 10     CONTINUE
C        NDN = 0
C       READ FROM INPUT FILE: NODE NUMBER AND SPECIFIED VALUE OF
C       FIELD VARIABLE
 20     READ(INF,*) I,X(I)
        IF (I .NE. -1) THEN
          IF(NDN .EQ. 0) WRITE(OUTF,30) LABEL1
 30       FORMAT(//3X,'NODE',15X,'SPECIFIED'/4X,'NO.',10X,A/
     1           2X,'------',9X,'--------------------')
          NDN = NDN + 1
          ICH(I) = 1
C         WRITE INFORMATION JUST READ TO OUTPUT FILE
          WRITE(OUTF,40) I,X(I)
 40       FORMAT(I6,10X,F15.4)
          GOTO 20
        ENDIF
        WRITE(OUTF,50) LABEL1,NDN
 50     FORMAT(//' # OF NODES WITH SPECIFIED ',A23,'=',I4)
        NNN = 0
C       READ FROM INPUT FILE: NODE NUMBER AND SPECIFIED VALUE OF
C       GROUNDWATER FLOW
 60     READ(INF,*) I,FLUX(I)
        IF (I .NE. -1) THEN
          IF (NNN .EQ. 0) WRITE(OUTF,30) LABEL2
          NNN = NNN + 1
C         WRITE THE INFORMATION JUST READ TO OUTPUT FILE
          WRITE(OUTF,40) I,FLUX(I)
          GOTO 60
        ENDIF
        WRITE(OUTF,50) LABEL2,NNN
        LCH(1) = ICH(1)
        DO 70 I = 2, NUMNOD
          LCH(I) = LCH(I-1) + ICH(I)
 70     CONTINUE
        NDOF = NUMNOD - NDN
        RETURN
        END
```

```
      SUBROUTINE INITIAL
C*********************************************************************
C
C         SUBROUTINE INITIAL INPUTS CONTROL PARAMETERS AND INITIAL
C         CONDITIONS NEEDED TO SOLVE TRANSIENT GROUNDWATER FLOW
C         PROBLEMS. SUBROUTINE INITIAL IS ALSO USED TO INPUT CONTROL
C         PARAMETERS AND TO SPECIFY INITIAL ESTIMATES FOR PRESSURE HEAD NEEDE
C         TO SOLVE STEADY-STATE,  UNSATURATED FLOW PROBLEMS.
C
C  INPUT:
C         ALL DATA ARE READ "FREE-FORMAT" FROM THE USER-SUPPLIED
C         FILE ASSIGNED TO UNIT "INF".  THE RELAXATION FACTOR
C         USED IN THE FINITE ELEMENT APPROXIMATION OF THE TIME
C         DERIVATIVE, A LIST OF TIME STEP INTERVALS, AND A LIST OF
C         VALUES OF THE TIME FUNCTION ARE ALSO READ.
C         THESE ARE FOLLOWED BY A LIST OF INITIAL VALUES OF THE
C         FIELD VARIABLE FOR EACH NODE IN THE MESH.
C
C  OUTPUT:
C         THE RELAXATION FACTOR, TIME STEP INTERVALS, VALUES OF
C         THE TIME FUNCTION, AND INITIAL VALUES OF THE FIELD
C         VARIABLE ARE WRITTEN TO THE USER-DEFINED FILE ASSIGNED
C         TO UNIT "OUTF".
C
C  DEFINITIONS OF VARIABLES:
C         DELTAT(I) = SIZE OF TIME STEP I
C         DTSTEP(I) = NUMBER OF TIME STEPS TO TAKE USING A TIME
C                     STEP OF SIZE DELTAT(I)
C             GT(I) = VALUE OF TIME FUNCTION AT TIME I
C            ICH(I) = 1 IF THE VALUE OF THE FIELD VARIABLE IS
C                     SPECIFIED FOR NODE I,
C                   = 0 OTHERWISE
C            LABEL1 = CHARACTER VARIABLE USED TO LABEL COLUMN
C                     HEADINGS FOR SPECIFIED VALUES OF THE FIELD
C                     VARIABLE ON FILE ASSIGNED TO UNIT OUTF.
C                     LABEL1 = "HYDRAULIC HEAD", "PRESSURE HEAD".
C                     MXSTEP = NUMBER OF DIFFERENT TIME STEP INTERVALS
C            NUMNOD = NUMBER OF NODES
C             OMEGA = RELAXATION FACTOR
C           OMOMEGA = 1. - OMEGA
C           TIME(I) = STARTING TIME FOR TIME FUNCTION VALUE GT(I)
C            TOTALT = TOTAL LENGTH OF TIME FOR WHICH CALCULATIONS
C                     ARE PERFORMED
C              X(I) = VALUE OF THE FIELD VARIABLE (HYDRAULIC
C                     HEAD, PRESSURE HEAD) AT NODE I
C
C   USAGE:
C         THE RELAXATION FACTOR OMEGA IS READ FIRST.  THIS
C         IS FOLLOWED BY A LIST OF TIME STEPS AND TIME STEP
C         INTERVALS. EACH LINE OF INPUT CONTAINS THE NUMBER OF
C         TIME STEPS TO TAKE FOLLOWED BY A SPECIFIED TIME STEP
C         INTERVAL.
C
 SUBROUTINES CALLED:
C         NONE
C*********************************************************************
       INCLUDE  'COMALL.IN'
C
C       INPUT OMEGA FROM INPUT FILE
        READ(INF,*) OMEGA
```

```
      WRITE(OUTF,10) OMEGA
 10   FORMAT(//2X,'OMEGA = ',F15.4)
      OMOMEGA = 1. - OMEGA
C     INPUT LIST OF TIME STEPS AND TIME STEP INTERVALS FROM INPUT FILE
      IT = 1
      MXSTEP = 0
 20   READ(INF,*) DTSTEP(IT),DELTAT(IT)
      IF (DTSTEP(IT) .LE. 0) GOTO 30
      IF (DTSTEP(IT) .GT. MXSTEP) MXSTEP = DTSTEP(IT)
      IT = IT + 1
      GOTO 20
 30   IT = IT - 1
      WRITE(OUTF,40)
 40   FORMAT(//2X,'START',8X,' END ',10X,'DELTA T'/
     1       2X,5('-'),8X,5('-'),8X,11('-'))
      ISTART = 1
      TOTALT = 0.
      DO 60 I = 1, IT
        WRITE(OUTF,50) ISTART,DTSTEP(I),DELTAT(I)
 50     FORMAT(2X,I4,9X,I4,3X,F15.4)
        TOTALT = TOTALT + (DTSTEP(I) - ISTART + 1) * DELTAT(I)
        ISTART = DTSTEP(I) + 1
 60   CONTINUE
      WRITE(OUTF,70) TOTALT
 70   FORMAT(/10X,'TOTAL TIME =',F15.4)
C     INPUT LIST OF TIME STEPS AND VALUES OF TIME FUNCTION
      IT = 1
 80   READ(INF,*) TIME(IT),GT(IT)
      IF (TIME(IT) .LT. 0.) GOTO 90
      IT = IT + 1
      GOTO 80
 90   IT = IT - 1
      IF (TIME(IT) .LT. TOTALT) TIME(IT) = TOTALT
      WRITE(OUTF,100)
 100  FORMAT(//8X,'TIME T',11X,'G(T)'/7X,8('-'),9X,6('-'))
      DO 120 I = 1, IT
        WRITE(OUTF,110) TIME(I),GT(I)
 110    FORMAT(2F15.4)
 120  CONTINUE
C     INPUT INITIAL VALUES OF FIELD VARIABLE FROM INPUT FILE
      ISTART = 1
 130  READ(INF,*) IT,HINIT
      IF (IT .LE. 0) GOTO 150
      IF (IT .GT. MAX1) IT = MAX1
      DO 140 I = ISTART, IT
        IF (ICH(I) .NE. 1) X(I) = HINIT
 140  CONTINUE
      ISTART = IT + 1
      IF (ISTART .LE. MAX1) GOTO 130
 150  WRITE(OUTF,160) LABEL1,LABEL1
 160  FORMAT(//2X,'INITIAL VALUES OF ',A/2X,38('-')//
     1       2X,'NODE NO.',10X,A/2X,8('-'),10X,20('-'))
      DO 180 I = 1, NUMNOD
        IF (ICH(I) .EQ. 0) THEN
          WRITE(OUTF,170) I,X(I),' '
        ELSE
          WRITE(OUTF,170) I,X(I),'*'
        ENDIF
 170    FORMAT(2X,I5,12X,F15.4,A)
 180  CONTINUE
      WRITE(OUTF,190)
```

```
 190    FORMAT(/23X,'* = SPECIFIED')
        RETURN
        END




        SUBROUTINE DUMP(LOOP,HDF,VLF)
C*********************************************************************C
C         TO WRITE CONTENTS OF ARRAYS TO USER-SUPPLIED DATA
C         FILES
C
C  INPUT:
C         CONTROL INFORMATION IS READ FROM USER-SUPPLIED FILE
C         ASSIGNED TO UNIT "INF".  FIRST LINE IS CODE INDICATING
C         WHICH ARRAYS ARE TO BE WRITTEN TO A OUTPUT FILE, THE
C         SECOND LINE IS THE NAME OF THE OUTPUT FILE.  THESE
C         TWO LINES CAN BE REPEATED AS OFTEN AS DESIRED.
C
C  OUTPUT:
C         CONTENTS OF ARRAYS ARE WRITTEN TO A SET OF OUTPUT
C         FILES.
C
C  DEFINITIONS OF VARIABLES:
C         FNAME = OUTPUT FILE NAME
C         ICODE = ARRAYS TO BE WRITTEN TO FNAME:
C               = 1, NODE NUMBERS AND COORDINATES
C               = 2, ELEMENT NUMBERS, TYPES, AND NODE NUMBERS
C               = 3, ELEMENT NUMBERS, MATERIAL SET NUMBERS,
C                    AND MATERIAL SET PROPERTIES
C               = 4, NODE NUMBERS AND SPECIFIED VALUES OF HEAD
C                    OR SOLUTE CONCENTRATION (DIRICHLET BOUNDARY
C                    CONDITIONS) AND SPECIFIED RATES OF GROUNDWATER
C                    FLOW.
C               = 5, RELAXATION FACTOR, TIME FUNCTION, AND INITIAL
C                    VALUES OF HEAD OR SOLUTE CONCENTRATION
C               = 6, COMPUTED VALUES OF HEAD OR SOLUTE CONCENTRATION
C               = 7, ELEMENT NUMBERS AND COMPONENTS OF APPARENT
C                    GROUNDWATER VELOCITY
C
C  USAGE:
C         THE CONTENTS OF THE ARRAYS ARE WRITTEN "FREE-FORMAT" TO
C         EACH DATA FILE.
C
C       SUBROUTINES CALLED:
C         NONE
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
        INTEGER DMPF,HDF,VLF
        LOGICAL LOOP,OPNED
        CHARACTER*20 FNAME
        DIMENSION XYZ(MAX1,3),V(MAX2,3),NODETBL(13)
        EQUIVALENCE (X1,XYZ(1,1)),(X2,XYZ(1,2)),(X3,XYZ(1,3)),
     1    (V1,V(1,1)),(V2,V(1,2)),(V3,V(1,3))
        DATA NODETBL/2,3,4,3,4,4,8,12,8,20,32,3,4/
C
        HDF = 0
        VLF = 0
 10     READ(INF,*,END=140,ERR=140) ICODE
```

```
      IF (ICODE .LE. 0) GOTO 140
      READ(INF,20,END=140,ERR=10) FNAME
 20   FORMAT(A)
      IF (ICODE .LE. 6) THEN
        DMPF = 1
      ELSE
        DMPF = 2
      ENDIF
      INQUIRE(UNIT=DMPF,OPENED=OPNED)
      IF (.NOT. OPNED)
     1  OPEN(DMPF,FILE=FNAME,STATUS='NEW',FORM='FORMATTED')
      IF (ICODE .EQ. 1) THEN
C
C     WRITE OUT NODE NUMBERS AND COORDINATES
C
        IF (DIM .LT. 4) THEN
          IDIM = DIM
        ELSE
          IDIM = 2
        ENDIF
        DO 30 I = 1, NUMNOD
          WRITE(DMPF,*) I,(XYZ(I,J),J=1,IDIM)
 30     CONTINUE
      ELSEIF (ICODE .EQ. 2) THEN
C
C     WRITE OUT ELEMENT NUMBERS, TYPES, AND NODE NUMBERS
C
        DO 40 I=1, NUMELM
    WRITE(DMPF,*) I,ELEMTYP(I),(IN(I,J),J=1,NODETBL(ELEMTYP(I)))
 40     CONTINUE
      ELSEIF (ICODE .EQ. 3) THEN
C
C     WRITE OUT ELEMENT AND MATERIAL SET NUMBERS AND MATERIAL
C     PROPERTIES
C
        DO 50 I = 1, NUMELM
          WRITE(DMPF,*) I,MATSET(I)
 50     CONTINUE
        WRITE(DMPF,*) NUMPROP
        DO 60 I = 1, NUMMAT
          WRITE(DMPF,*) I,(PROP(I,J),J=1,NUMPROP)
 60     CONTINUE
      ELSEIF (ICODE .EQ. 4) THEN
C
C     WRITE OUT SPECIFED VALUES OF FIELD VARIABLE AND GROUNDWATER
C     FLOW OR SOLUTE FLUX
C
        IF (NDN .GT. 0) THEN
          DO 70 I = 1, NUMNOD
            IF (ICH(I) .NE. 0) WRITE(DMPF,*) I,X(I)
 70       CONTINUE
        ENDIF
        IF (NNN .GT. 0) THEN
          DO 80 I = 1, NUMNOD
            IF (FLUX(I) .NE. 0.) WRITE(DMPF,*) I,FLUX(I)
 80       CONTINUE
        ENDIF
      ELSEIF (ICODE .EQ. 5) THEN
C
C     WRITE OUT RELAXATION FACTOR, TIME FUNCTION, AND INITIAL
C     VALUES OF FIELD VARIABLE
```

```
C
          IF (LOOP) THEN
            WRITE(DMPF,*) OMEGA
            ISTART = 1
            IT = 1
            TOTALT = 0.
 90         WRITE(DMPF,*) ISTART,DTSTEP(IT),DELTAT(IT)
            TOTALT = TOTALT + (DTSTEP(IT) - ISTART + 1) * DELTAT(IT)
            IF (DTSTEP(IT) .LT. MXSTEP) THEN
              ISTART = DTSTEP(IT) + 1
              IT = IT + 1
              GOTO 90
            ELSE
              WRITE(DMPF,*) TOTALT
              IT = 1
 100          WRITE(DMPF,*) TIME(IT),GT(IT)
              IF (TIME(IT) .GE. 0. .AND. TIME(IT) .LT. TOTALT) THEN
                IT = IT + 1
                GOTO 100
              ELSE
                DO 110 I = 1, NUMNOD
                  IF (ICH(I) .EQ. 0) WRITE(DMPF,*) I,X(I)
 110            CONTINUE
              ENDIF
            ENDIF
          ENDIF
        ELSEIF (ICODE .EQ. 6) THEN
C
C       WRITE OUT COMPUTED VALUES OF FIELD VARIABLE
C
C
          IF (LOOP) THEN
            HDF = DMPF
          ELSE
            IF (DIM .LT. 4) THEN
             IDIM = DIM
            ELSE
             IDIM = 2
            ENDIF
            DO 120 I = 1, NUMNOD
              WRITE(DMPF,*) I,X(I)
 120        CONTINUE
          ENDIF
        ELSEIF (ICODE .EQ. 7) THEN
C
C       WRITE OUT ELEMENT NUMBERS AND COMPUTED COMPONENTS
C       OF APPARENT GROUNDWATER VELOCITY
C
          IF (LOOP) THEN
            VLF = DMPF
          ELSE
            IF (DIM .LT. 4) THEN
              IDIM = DIM
            ELSE
              IDIM = 2
            ENDIF
            DO 130 I = 1, NUMELM
              WRITE(DMPF,*) I,(V(I,J),J=1,IDIM)
 130        CONTINUE
          ENDIF
        ENDIF
```

```
      IF (ICODE .LE. 5 .OR. (ICODE .GT. 5 .AND. .NOT. LOOP))
     1   CLOSE(UNIT=DMPF)
      GOTO 10
 140  RETURN
      END
```

```
      SUBROUTINE ASMBKC
C*********************************************************************
C
C         TO ASSEMBLE THE COMBINED GLOBAL CONDUCTANCE AND
C         CAPACITANCE MATRIX AND THE  GLOBAL SPECIFIED FLOW MATRIX
C         FOR THE MESH AND TO MODIFY THE SYSTEM OF EQUATIONS FOR
C         SPECIFIED HEAD AND GROUNDWATER FLOW BOUNDARY CONDITIONS
C
C  INPUT:
C         NONE
C
C  OUTPUT:
C         THE SEMI-BANDWIDTH AND NUMBER OF DEGREES OF FREEDOM
C         FOR THE MODIFIED, COMBINED GLOBAL CONDUCTANCE AND
C         CAPACITANCE MATRIX ARE WRITTEN TO THE USER-DEFINED
C         FILE ASSIGNED TO UNIT "OUTF"
C
C  DEFINITIONS OF VARIABLES:
C                         B(I) = GLOBAL GROUNDWATER FLOW MATRIX
C                      CE(I,J) = CAPACITANCE MATRIX FOR ELEMENT E IN
C                                FULL MATRIX STORAGE
C                            E = ELEMENT NUMBER
C                   ELEMTYP(E) = ELEMENT TYPE FOR ELEMENT E
C                      FLUX(I) = SPECIFIED RATE OF GROUNDWATER FLOW
C                                AT NODE I
C                       ICH(I) = 1 IF THE VALUE OF HYDRAULIC HEAD OR
C                                PRESSURE HEAD IS SPECIFIED FOR NODE I,
C                              = 0 OTHERWISE
C                       IJSIZE = LENGTH OF ARRAY [M] IN VECTOR STORAGE
C                      KE(I,J) = CONDUCTANCE MATRIX FOR ELEMENT E IN
C                                FULL MATRIX STORAGE
C                       LCH(I) = ICH(I) + ICH(I-1) + ICH(I-2) + ...
C                                THE ARRAYS ICH AND LCH ARE USED TO
C                                MODIFY THE GLOBAL MATRIX
C                        M(IJ) = MODIFIED, COMBINED GLOBAL CONDUCTANCE
C                                AND CAPACITANCE MATRIX IN VECTOR
C                                STORAGE
C                         NDOF = NUMBER OF NODES WHERE THE VALUE OF
C                                THE FIELD VARIABLE IS UNKNOWN
C         NODETBL(ELEMTYP(E)) = NUMBER OF NODES IN ELEMENT TYPE E
C                       NUMELM = NUMBER OF ELEMENTS IN MESH
C                          SBW = SEMI-BANDWIDTH OF MODIFIED, COMBINED
C                                GLOBAL CONDUCTANCE AND CAPACITANCE
C                                MATRIX
C                         X(I) = VALUE OF HYDRAULIC HEAD OR PRESSURE
C                                HEAD AT NODE I
C
C  USAGE:
C         THE SEMI-BANDWIDTH OF THE COMBINED GLOBAL CONDUCTANCE AND
C         CAPACITANCE MATRIX IS COMPUTED FIRST.  THEN THE ENTRIES
C         OF THE ELEMENT CONDUCTANCE AND CAPACITANCE MATRICES ARE
C         COMPUTED IN A SET OF SUBROUTINES, TWO SUBROUTINES FOR
C         EACH ELEMENT TYPE.  THE COMBINED GLOBAL CONDUCTANCE AND
```

```
C         CAPACITANCE MATRIX FOR THE MESH IS ASSEMBLED BY ADDING THE
C         CORRESPONDING ENTRIES OF THE ELEMENT MATRICES TO THE GLOBAL
C         MATRIX.  DURING THE ASSEMBLY PROCESS THE GLOBAL MATRIX IS
C         MODIFIED FOR SPECIFIED VALUES OF HEAD AND SPECIFIED VALUES
C         OF GROUNDWATER FLOW ARE ADDED TO THE GLOBAL FLOW MATRIX.
C
C       SUBROUTINES CALLED:
C         KBAR2,KTRI3,KREC4,KQUA4,LOC
C         CBAR2,CTRI3,CREC4,CQUA4
C
C
C**********************************************************************
        INCLUDE  'COMALL.IN'
        REAL KE(MAX3,MAX3),CE(MAX3,MAX3)
        INTEGER NODETBL(13)
        DATA NODETBL/2,3,4,3,4,4,8,12,8,20,32,3,4/
C
C       COMPUTE THE SEMI-BANDWIDTH
        SBW = 1
        DO 30 E = 1, NUMELM
          DO 20 I = 1, NODETBL(ELEMTYP(E))
            KI = IN(E,I)
            IF (ICH(KI) .EQ. 0 .AND. I .LT. NODETBL(ELEMTYP(E))) THEN
              II = KI - LCH(KI)
              DO 10 J = I + 1, NODETBL(ELEMTYP(E))
                KJ = IN(E,J)
                IF (ICH(KJ) .EQ. 0) THEN
                  JJ = ABS(KJ - LCH(KJ) - II) + 1
                  IF (JJ .GT. SBW) SBW = JJ
                ENDIF
 10           CONTINUE
            ENDIF
 20       CONTINUE
 30     CONTINUE
C        WRITE(OUTF,40) NDOF,SBW
C 40     FORMAT(//' NUMBER OF DEGREES OF FREEDOM IN MODIFIED,'/
C     1         ' GLOBAL COMBINED CONDUCTANCE AND CAPCAITANCE',
C     2         ' MATRIX =',I5///' SEMI-BANDWIDTH OF MODIFIED,'/
C     3         ' GLOBAL COMBINED CONDUCTANCE AND CAPCAITANCE',
C     4         ' MATRIX =',I5)
        IF (SBW .GT. MAX6) STOP'** EXCEEDS MAXIMUM SEMI-BAND WIDTH **'
C       INITIALIZE ENTRIES OF GLOBAL MATRIX TO ZERO
        IJSIZE = SBW * (NDOF - SBW + 1) + (SBW - 1) * SBW / 2
        DO 50 IJ = 1, IJSIZE
          M(IJ) = 0.0
          B1(IJ) = 0.0
 50     CONTINUE

        DO 56 I = 1, MAX1
 56       FC(I) = 0.

C       INITIALIZE ENTRIES OF THE GLOBAL GROUNDWATER MATRIX TO ZERO
        DO 60 I = NDOF
          B(I) = 0.0
 60     CONTINUE
C       LOOP ON THE NUMBER OF ELEMENTS
        DO 90 E = 1, NUMELM
C         COMPUTE THE ELEMENT CONDUCTANCE AND CAPACITANCE MATRICES
C         FOR THIS ELEMENT TYPE
          IF (ELEMTYP(E) .EQ. 1) THEN
C           ELEMENT IS A ONE-DIMENSIONAL, LINEAR BAR
```

```
            CALL KBAR2(E,KE)
            CALL CBAR2(E,CE)
          ELSEIF (ELEMTYP(E) .EQ. 4) THEN
C           ELEMENT IS A TWO-DIMENSIONAL, LINEAR TRIANGLE
            CALL KTRI3(E,KE)
            CALL CTRI3(E,CE)
          ELSEIF (ELEMTYP(E) .EQ. 5) THEN
C           ELEMENT IS A TWO-DIMENSIONAL, LINEAR RECTANGLE
            CALL KREC4(E,KE)
            CALL CREC4(E,CE)
          ELSEIF (ELEMTYP(E) .EQ. 6) THEN
C           ELEMENT IS A TWO-DIMENSIONAL, LINEAR QUADRILATERAL
            CALL KQUA4(E,KE)
            CALL CQUA4(E,CE)

          ENDIF
C         ADD THE ELEMENT CONDUCTANCE AND CAPACITANCE MATRICES FOR
C         THIS ELEMENT TO THE GLOBAL MATRIX
C    KE(I,J),CE(I,J) ----------->    M(IJ)     <=>            M(KI,KJ)
C (FULL MATRIX STORAGE)     (VECTOR MATRIX STORAGE)     (FULL MATRIXSTORAGE)
          DO 80 I = 1, NODETBL(ELEMTYP(E))
            KI = IN(E,I)
            IF (ICH(KI) .EQ. 0) THEN
              II = KI - LCH(KI)
              DO 70 J = 1, NODETBL(ELEMTYP(E))
                KJ = IN(E,J)
                IF (ICH(KJ) .NE. 0) THEN
                  FC(II) = FC(II) - DELTAT(IDT) * KE(I,J) * X(KJ)
                ELSEIF (J .GE. I) THEN
                  JJ = KJ - LCH(KJ)
                  CALL LOC(II,JJ,IJ,NDOF,SBW,SYMM)
                  M(IJ) = M(IJ) + CE(I,J) + OMEGA *
     1                    DELTAT(IDT) * KE(I,J)
                  B1(IJ) = B1(IJ) + CE(I,J) - OMOMEGA *
     1                      DELTAT(IDT) * KE(I,J)
                ENDIF
 70           CONTINUE
            ENDIF
 80       CONTINUE
 90     CONTINUE
        RETURN
        END




        SUBROUTINE LOC(I,J,IJ,NDOF,SBW,SYMM)
C*************************************************************
C
C         SUBROUTINE LOC COMPUTES THE LOCATION IN VECTOR STORAGE
C         OF A SPECIFIED ROW AND COLUMN OF A MATRIX (SYMMETRIC OR
C         NONSYMMETRIC) IN FULL MATRIX STORAGE
C
C       DEFINITIONS OF VARIABLES:
C            I = SPECIFIED ROW OF MATRIX IN FULL MATRIX STORAGE
C            J = SPECIFIED COLUMN OF MATRIX IN FULL MATRIX
C                STORAGE
C           IJ = LOCATION IN VECTOR STORAGE CORRESPONDING TO
C                SPECIFIED ROW AND COLUMN IN FULL MATRIX STORAGE
C         NDOF = NUMBER OF DEGREES OF FREEDOM OF MATRIX
C          SBW = SEMI-BANDWIDTH OF MATRIX
```

```
C
C*************************************************************

        INTEGER SBW
        LOGICAL SYMM
C
        IF (SYMM) THEN
C         SYMMETRIC MATRIX
          II = I
          JJ = J
          IF (I .GT. J) THEN
            K = I
            I = J
            J = K
          ENDIF
          IJ = J - I + 1
          IF (I .GT. 1) THEN
            IF (SBW .LT. NDOF) THEN
              IJ = IJ + (I - 1) * SBW
              L = I - NDOF + SBW - 2
              IF (L .GT. 0) IJ = IJ - L * (L + 1) / 2
            ELSE
              IJ = IJ + (I - 1) * (NDOF + (NDOF - I + 2)) / 2
            ENDIF
          ENDIF
          I = II
          J = JJ
        ELSE
C         NONSYMMETRIC MATRIX
          IJ = J
          IF (I .GT. 1) THEN
            IF (SBW .LT. NDOF) THEN
              IF (I .GT. SBW) IJ = IJ + SBW - I
              IJ = IJ + (I - 1) * (2 * SBW - 1)
              L = MIN(SBW,I) - 1
              IJ = IJ - L * ((SBW - 1) + (SBW - L)) / 2
              L = I - NDOF + SBW -2
              IF (L .GT. 0) IJ = IJ - L * (L + 1) / 2
            ELSE
              IJ = IJ + (I - 1) * NDOF
            ENDIF
          ENDIF
        ENDIF
        RETURN
        END



      SUBROUTINE CBAR2(E,CE)
C**********************************************************************
C
C         TO COMPUTE THE CONSISTENT FORM OF THE ELEMENT
C         CAPACITANCE MATRIX FOR A ONE-DIMENSIONAL, LINEAR
C         BAR ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               E = ELEMENT NUMBER
C         CE(I,J) = ELEMENT CAPACITANCE MATRIX
C             SSE = ELEMENT SPECIFIC STORAGE
C              LE = ELEMENT LENGTH
C
C*****************************************************************
        INCLUDE  'COMALL.IN'
```

```
        REAL CE(MAX3,MAX3),LE
C
        SSE = PROP(E,2)
        LE = ABS(X1(IN(E,2)) - X1(IN(E,1)))
        CE(1,1) = SSE * LE / 3.
        CE(1,2) = SSE * LE / 6.
        CE(2,1) = CE(1,2)
        CE(2,2) = CE(1,1)
        RETURN
        END
```

```
      SUBROUTINE CTRI3(E,CE)
C*****************************************************************
C
C         TO COMPUTE THE CONSISTENT FORM OF THE ELEMENT CAPACITANCE
C         MATRIX FOR TWO- DIMENSIONAL, LINEAR TRIANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C             AE4 = FOUR TIMES ELEMENT AREA
C               E = ELEMENT NUMBER
C         CE(I,J) = ELEMENT CAPACITANCE MATRIX
C             SSE = ELEMENT SPECIFIC STORAGE
C
C*****************************************************************
       INCLUDE  'COMALL.IN'
      REAL CE(MAX3,MAX3)
C
       SSE = PROP(E,3)
       AE4 = 2 * (X1(IN(E,2)) * X2(IN(E,3)) + X1(IN(E,1)) *
     1       X2(IN(E,2)) +  X2(IN(E,1)) * X1(IN(E,3)) -
     2       X2(IN(E,3)) * X1(IN(E,1)) - X1(IN(E,3)) *
     3       X2(IN(E,2)) - X1(IN(E,2)) * X2(IN(E,1)))
       AE = AE4 / 4.
       CE(1,1) = SSE * AE / 6.
       CE(1,2) = CE(1,1) / 2.
       CE(1,3) = CE(1,2)
       CE(2,1) = CE(1,2)
       CE(2,2) = CE(1,1)
       CE(2,3) = CE(1,2)
       CE(3,1) = CE(1,2)
       CE(3,2) = CE(1,2)
       CE(3,3) = CE(1,1)
       RETURN
       END
```

```
      SUBROUTINE CREC4(E,CE)
C*****************************************************************
C
C         TO COMPUTE THE CONSISTENT FORM OF THE ELEMENT CAPACITANCE
C         MATRIX FOR TWO- DIMENSIONAL, LINEAR TRIANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               E = ELEMENT NUMBER
C         CE(I,J) = ELEMENT CAPACITANCE MATRIX
```

```
C              SSE = ELEMENT SPECIFIC STORAGE
C
C*********************************************************************
      INCLUDE  'COMALL.IN'
      REAL CE(MAX3,MAX3)
C
       SSE = PROP(E,3)
       AE = ABS(X2(IN(E,1)) - X2(IN(E,3))) / 2.
       BE = ABS(X1(IN(E,1)) - X1(IN(E,3))) / 2.
       TEMP = (SSE * AE * BE) / 9
       CE(1,1) = 4. * TEMP
       CE(1,2) = 2. * TEMP
       CE(1,3) = TEMP
       CE(1,4) = CE(1,2)
       CE(2,1) = CE(1,2)
       CE(2,2) = CE(1,1)
       CE(2,3) = CE(1,2)
       CE(2,4) = CE(1,3)
       CE(3,1) = CE(1,3)
       CE(3,2) = CE(1,2)
       CE(3,3) = CE(1,1)
       CE(3,4) = CE(1,2)
       CE(4,1) = CE(1,2)
       CE(4,2) = CE(1,3)
       CE(4,3) = CE(1,2)
       CE(4,4) = CE(1,1)
       RETURN
       END




      SUBROUTINE CQUA4(E,CE)
C*********************************************************************
C
C         TO COMPUTE THE CONSISTENT FORM OF THE ELEMENT CAPACITANCE
C         MATRIX FOR A TWO-DIMENSIONAL, LINEAR QUADRILATERAL ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C             CE(I,J) = ELEMENT CAPACITANCE MATRIX
C              DETJAC = DETERMINANT OF JACOBIAN MATRIX
C            DNDXI(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO XI AT NODE I
C             DNDX(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO X AT NODE I
C           DNDETA(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH ESPECT TO ETA AT NODE I
C             DNDY(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO Y AT NODE I
C               XI(I) = LOCATION OF GAUSS POINT IN XI COORDINATE
C                       DIRECTION
C              ETA(I) = LOCATION OF GAUSS POINT IN ETA COORDINATE
C                       DIRECTION
C            JAC(I,J) = JACOBIAN MATRIX
C                   E = ELEMENT NUMBER
C                 SSE = ELEMENT SPECIFIC STORAGE
C                N(I) = INTERPOLATION FUNCTION FOR NODE I
C                W(I) = WEIGHT FOR GAUSS POINT I
C           X1(IN(E,I) = X COORDINATE FOR NODE I, ELEMENT E
C           X2(IN(E,I) = Y COORDINATE FOR NODE I, ELEMENT E
```

```
C
C*****************************************************************
       INCLUDE  'COMALL.IN'
      REAL JAC(2,2),JACINV(2,2),CE(MAX3,MAX3),N(4),DNDXI(4),
     1     DNDR(4),DNDETA(4),DNDZ(4),W(2),XI(2),ETA(2),SIGN1(4),
     2     SIGN2(4)

      DATA SIGN1/-1.,1.,1.,-1./
      DATA SIGN2/-1.,-1.,1.,1./
C
      XI(1) =  1. / SQRT(3.)
      XI(2) = -XI(1)
      ETA(1) = XI(1)
      ETA(2) = XI(2)
      W(1) = 1.
      W(2) = 1.
      SSE = PROP(E,3)

      DO 30 I = 1, 4
        DO 20 J = 1, 4
          CE(I,J) = 0.
 20     CONTINUE
 30   CONTINUE

      DO 120 I = 1, 2
        DO 110 J = 1, 2
          DO 50 K = 1, 2
            DO 40 K1 = 1, 2
              JAC(K,K1) = 0.
 40         CONTINUE
 50       CONTINUE

          DO 60 K1 = 1, 4
                 N(K1) = 0.25 * (1. + SIGN1(K1) * XI(I))
     1                        * (1. + SIGN2(K1) * ETA(J))
             DNDXI(K1) = 0.25 * SIGN1(K1) * (1. + SIGN2(K1) * ETA(J))
            DNDETA(K1) = 0.25 * SIGN2(K1) * (1. + SIGN1(K1) * XI(I))
 60       CONTINUE
          DO 70 K1 = 1, 4
            JAC(1,1) = JAC(1,1) + DNDXI(K1) * X1(IN(E,K1))
            JAC(1,2) = JAC(1,2) + DNDXI(K1) * X2(IN(E,K1))
            JAC(2,1) = JAC(2,1) + DNDETA(K1) * X1(IN(E,K1))
            JAC(2,2) = JAC(2,2) + DNDETA(K1) * X2(IN(E,K1))
 70       CONTINUE
          DETJAC = JAC(1,1) * JAC(2,2) - JAC(1,2) * JAC(2,1)
          DO 100 K = 1, 4
            DO 90 K1 = 1, 4
              CE(K,K1) = CE(K,K1) + W(I) * W(J) * SSE* N(K) *
     1                   N(K1) * DETJAC
 90         CONTINUE
 100      CONTINUE
 110    CONTINUE
 120  CONTINUE
      RETURN
      END




      SUBROUTINE KBAR2(E,KE)
C*********************************************************************
```

```
C
C         TO COMPUTE THE SATURATED FORM OF THE ELEMENT
C         CONDUCTANCE MATRIX FOR A ONE-DIMENSIONAL, LINEAR
C         BAR ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               E = ELEMENT NUMBER
C         KE(I,J) = ELEMENT CONDUCTANCE MATRIX
C             KXE = HYDRAULIC CONDUCTIVITY IN X COORDINATE DIRECTION
C              LE = ELEMENT LENGTH
C
C**********************************************************************
         INCLUDE  'COMALL.IN'
        REAL KE(MAX3,MAX3),KXE,LE
C
        KXE = PROP(E,1)
        LE = ABS(X1(IN(E,2)) - X1(IN(E,1)))
        KE(1,1) =  KXE / LE
        KE(1,2) = -KE(1,1)
        KE(2,1) = -KE(1,1)
        KE(2,2) =  KE(1,1)
        RETURN
        END




       SUBROUTINE KTRI3(E,KE)
C**********************************************************************
C
C         TO COMPUTE THE ELEMENT CONDUCTANCE MATRIX FOR TWO-
C         DIMENSIONAL, LINEAR TRIANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C             AE4 = FOUR TIMES ELEMENT AREA
C               E = ELEMENT NUMBER
C         KE(I,J) = ELEMENT CONDUCTANCE MATRIX
C             KXE = HYDRAULIC CONDUCTIVITY IN X
C                   COORDINATE DIRECTION
C             KYE = HYDRAULIC CONDUCTIVITY IN Y
C                   COORDINATE DIRECTION
C
C**********************************************************************
       INCLUDE  'COMALL.IN'
      REAL KE(MAX3,MAX3),KXE,KYE,BE(3),CE(3)
C
      KXE = PROP(E,1)
      KYE = PROP(E,2)
      BE(1) = X2(IN(E,2)) - X2(IN(E,3))
      BE(2) = X2(IN(E,3)) - X2(IN(E,1))
      BE(3) = X2(IN(E,1)) - X2(IN(E,2))
      CE(1) = X1(IN(E,3)) - X1(IN(E,2))
      CE(2) = X1(IN(E,1)) - X1(IN(E,3))
      CE(3) = X1(IN(E,2)) - X1(IN(E,1))
      AE4 = 2 * (X1(IN(E,2)) * X2(IN(E,3)) + X1(IN(E,1)) *
     1      X2(IN(E,2)) +  X2(IN(E,1)) * X1(IN(E,3)) -
     2      X2(IN(E,3)) * X1(IN(E,1)) - X1(IN(E,3)) *
     3      X2(IN(E,2)) - X1(IN(E,2)) * X2(IN(E,1)))
      DO 20 I = 1, 3
        DO 10 J = 1, 3
```

```
          KE(I,J) = (KXE * BE(I) * BE(J) + KYE * CE(I) * CE(J)) / AE4
 10     CONTINUE
 20   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE KREC4(E,KE)
C*********************************************************************
C
C         TO COMPUTE THE ELEMENT CONDUCTANCE MATRIX FOR TWO-
C         DIMENSIONAL, LINEAR RECTANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               E = ELEMENT NUMBER
C         KE(I,J) = ELEMENT CONDUCTANCE MATRIX
C             KXE = HYDRAULIC CONDUCTIVITY IN X
C                   COORDINATE DIRECTION
C             KYE = HYDRAULIC CONDUCTIVITY IN Y
C                   COORDINATE DIRECTION
C
C*********************************************************************
       INCLUDE  'COMALL.IN'
      REAL KE(MAX3,MAX3),KXE,KYE
C
      KXE = PROP(E,1)
      KYE = PROP(E,2)
      AE = ABS(X2(IN(E,1))-X2(IN(E,3))) / 2.
      BE = ABS(X1(IN(E,1)) - X1(IN(E,3))) / 2.
      CX = KXE * AE / (6. * BE)
      CY = KYE * BE / (6. * AE)
      KE(1,1) =  2. * CX + 2. * CY
      KE(1,2) = -2. * CX + CY
      KE(1,3) = -CX - CY
      KE(1,4) =  CX - 2. * CY
      KE(2,1) = KE(1,2)
      KE(2,2) =  2. * CX + 2. * CY
      KE(2,3) =  CX - 2. * CY
      KE(2,4) = -CX - CY
      KE(3,1) = KE(1,3)
      KE(3,2) = KE(2,3)
      KE(3,3) =  2. * CX + 2. * CY
      KE(3,4) = -2. * CX + CY
      KE(4,1) = KE(1,4)
      KE(4,2) = KE(2,4)
      KE(4,3) = KE(3,4)
      KE(4,4) = 2. * CX + 2. * CY
      RETURN
      END
```

```
      SUBROUTINE KQUA4(E,KE)
C*********************************************************************
C
C         TO COMPUTE THE ELEMENT CONDUCTANCE MATRIX FOR TWO-
C         DIMENSIONAL, LINEAR QUADRILATERAL ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C              DETJAC = DETERMINANT OF JACOBIAN MATRIX
C            DNDXI(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO XI AT NODE I
C             DNDX(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO X AT NODE I
C           DNDETA(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH ESPECT TO ETA AT NODE I
C             DNDY(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO Y AT NODE I
C                   E = ELEMENT NUMBER
C              ETA(I) = LOCATION OF GAUSS POINT IN ETA COORDINATE
C                       DIRECTION
C            JAC(I,J) = JACOBIAN MATRIX
C         JACINV(I,J) = INVERSE OF JACOBIAN MATRIX
C             KE(I,J) = ELEMENT CONDUCTANCE MATRIX
C                 KXE = HYDRAULIC CONDUCTIVITY IN X
C                       COORDINATE DIRECTION
C                 KYE = HYDRAULIC CONDUCTIVITY IN Y
C                       COORDINATE DIRECTION
C                W(I) = WEIGHT FOR GAUSS POINT I
C          X1(IN(E,I) = X COORDINATE FOR NODE I, ELEMENT E
C          X2(IN(E,I) = Y COORDINATE FOR NODE I, ELEMENT E
C               XI(I) = LOCATION OF GAUSS POINT IN XI COORDINATE
C                       DIRECTION
C*****************************************************************
       INCLUDE  'COMALL.IN'
      REAL JAC(2,2),JACINV(2,2),KE(MAX3,MAX3),DNDXI(4),DNDX(4),
     1     DNDETA(4),DNDY(4),W(2),XI(2),ETA(2),SIGN1(4),SIGN2(4),
     2     KXE,KYE
      DATA SIGN1/-1.,1.,1.,-1./
      DATA SIGN2/-1.,-1.,1.,1./
C
      XI(1) =  1. / SQRT(3.)
      XI(2) = -XI(1)
      ETA(1) = XI(1)
      ETA(2) = XI(2)
      W(1) = 1.
      W(2) = 1.
      KXE = PROP(E,1)
      KYE = PROP(E,2)

      DO 30 K = 1, 4
        DO 20 N = 1, 4
          KE(K,N) = 0.
 20     CONTINUE
 30   CONTINUE

      DO 120 I = 1, 2
        DO 110 J = 1, 2

          DO 50 K = 1, 2
            DO 40 N = 1, 2
              JAC(K,N) = 0.
 40         CONTINUE
 50       CONTINUE
```

```
        DO 60 N = 1, 4
          DNDXI(N)  = 0.25 * SIGN1(N) * (1. + SIGN2(N) * ETA(J))
          DNDETA(N) = 0.25 * SIGN2(N) * (1. + SIGN1(N) * XI(I))
 60     CONTINUE
        DO 70 N = 1, 4
          JAC(1,1) = JAC(1,1) + DNDXI(N) * X1(IN(E,N))
          JAC(1,2) = JAC(1,2) + DNDXI(N) * X2(IN(E,N))
          JAC(2,1) = JAC(2,1) + DNDETA(N) * X1(IN(E,N))
          JAC(2,2) = JAC(2,2) + DNDETA(N) * X2(IN(E,N))
 70     CONTINUE
        DETJAC = JAC(1,1) * JAC(2,2) - JAC(1,2) * JAC(2,1)
        JACINV(1,1) =  JAC(2,2) / DETJAC
        JACINV(1,2) = -JAC(1,2) / DETJAC
        JACINV(2,1) = -JAC(2,1) / DETJAC
        JACINV(2,2) =  JAC(1,1) / DETJAC
        DO 80 N = 1, 4
          DNDX(N) = JACINV(1,1) * DNDXI(N) + JACINV(1,2) * DNDETA(N)
          DNDY(N) = JACINV(2,1) * DNDXI(N) + JACINV(2,2) * DNDETA(N)
 80     CONTINUE
        DO 100 K = 1, 4
          DO 90 N = 1, 4
            KE(K,N) = KE(K,N) + W(I) * W(J) * (KXE * DNDX(K) *
     1                DNDX(N) + KYE * DNDY(K) * DNDY(N)) * DETJAC
 90       CONTINUE
 100    CONTINUE
 110  CONTINUE
 120  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DECOMP(NDOF,SBW,SYMM,M)
C**********************************************************************
C
C         SUBROUTINES DECOMP AND SOLVE SOLVE A SYSTEM OF
C         LINEAR EQUATIONS OF THE FORM
C                     [M] {X} = {B}
C         WHERE [M] IS A BANDED MATRIX OF KNOWN COEFFICIENTS
C         (SYMMETRIC OR NONSYMMETRIC), {X} ARE THE UNKNOWNS,
C         AND {B} IS A VECTOR OF KNOWN VALUES
C
C  INPUT:
C         NONE
C
C  OUTPUT:
C         NONE
C
C  DEFINITIONS OF VARIABLES:
C          B(I) = VECTOR OF KNOWN VALUES
C         M(IJ) = MATRIX OF KNOWN COEFFICIENTS IN VECTOR STORAGE
```

```
C           NDOF = NUMBER OF UNKNOWN VALUES IN {X}
C            SBW = SEMI-BANDWIDTH OF [M]
C           SYMM = LOGICAL VARIABLE
C                = 'TRUE' IF [M] IS SYMMETRIC
C                = 'FALSE' IF [M] IS NONSYMMETRIC
C           X(I) = VECTOR OF UNKNOWN VALUES TO BE COMPUTED
C
C  USAGE:
C          SUBROUTINE DECOMP PERFORMS TRIANGULAR DECOMPOSITION
C          ON THE MATRIX OF KNOWN COEFFICIENTS IN VECTOR MATRIX
C          STORAGE, {M}.  THE RESULTING UPPER-, AND LOWER-
C          TRIANGULAR MATRICES ARE STORED IN {M} (THE ORIGINAL
C          CONTENTS OF {M} ARE OVERWRITTEN DURING THE
C          DECOMPOSITION PROCESS).  SUBROUTINE SOLVE SOLVES FOR
C          VALUES OF THE UNKNOWNS BY BACKWARD SUBSTITUTION.  ONCE
C          {M} HAS BEEN DECOMPOSED SOLVE CAN BE USED TO OBTAIN
C          VALUES OF {X} FOR ANY NUMBER OF DIFFERENT VECTORS {B}.
C
C       SUBROUTINES CALLED:
C         LOC
C
C*********************************************************************
        INTEGER NDOF,SBW
        LOGICAL SYMM
        REAL M(1)
C
        IF (SYMM) THEN
C         M IS A SYMMETRIC MATRIX
          J2 = SBW
          IJ = 0
          DO 30 I = 1, NDOF
            II = IJ + 1
            DO 20 J = I, J2
              IJ = IJ + 1
              IF (I .GT. 1) THEN
                K1 = J - SBW + 1
                IF (K1 .LT. I) THEN
                  IF (K1 .LE. 0) K1 = 1
                  DO 10 K = K1, (I-1)
                    CALL LOC(K,I,KI,NDOF,SBW,SYMM)
                    CALL LOC(K,J,KJ,NDOF,SBW,SYMM)
                    M(IJ) = M(IJ) - M(KI) * M(KJ)
 10               CONTINUE
                ENDIF
              ENDIF
              IF (I .EQ. J) THEN
                M(IJ) = SQRT(M(IJ))
              ELSE
               M(IJ) = M(IJ) / M(II)
              ENDIF
 20         CONTINUE
            IF (J2 .LT. NDOF) J2 = J2 + 1
 30       CONTINUE
        ELSE
C         M IS A NONSYMMETRIC MATRIX
          J1 = 1
          J2 = SBW
          IJ = 0
          DO 60 I = 1, NDOF
            II = IJ + I - J1 + 1
            K1 = J1
```

```
          IKBEG = IJ + 1
          DO 50 J = J1, J2
            IJ = IJ + 1
            IF (J .GT. SBW .AND. I .LT. J) THEN
              K1 = K1 + 1
              IKBEG = IKBEG + 1
            ENDIF
            K2 = MIN(I,J) - 1
            IF (K2 .GE. K1) THEN
              IK = IKBEG
              DO 40 K = K1, K2
                CALL LOC(K,J,KJ,NDOF,SBW,SYMM)
                M(IJ) = M(IJ) - M(IK) * M(KJ)
                IK = IK + 1
40            CONTINUE
            ENDIF
            IF (I .LT. J) THEN
              M(IJ) = M(IJ) / M(II)
            ENDIF
50        CONTINUE
          IF (I .GE. SBW) J1 = J1 + 1
          IF (J2 .LT. NDOF) J2 = J2 + 1
60      CONTINUE
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE RHS
C*********************************************************************
C
C         SUBROUTINE RHS ASSEMBLES THE RIGHT-HAND-SIDE VECTOR
C         FOR TRANSIENT GROUNDWATER FLOW AND SOLUTE TRANSPORT
C         PROBLEMS.
C
C  INPUT:
C         NONE
C
C  OUTPUT:
C         NONE
C
C  DEFINITIONS OF VARIABLES:
C         DELTAT(I) = SIZE OF TIME STEP I
C           FLUX(I) = SPECIFIED VALUE OF GROUNDWATER FLOW OR AT NODE I
C             GT(I) = VALUE OF TIME FUNCITON AT TIME I
C            ICH(I) = 1 IF THE VALUE OF THE FIELD VARIABLE IS
C                     SPECIFIED AT NODE I
C                   = 0 OTHERWISE
C            B1(IJ) = MODIFIED GLOBAL MATRIX IN VECTOR STORAGE
C              NDOF = NUMBER OF NODES WHERE THE VALUE OF THE
C                     FIELD VARIABLE IS UNKNOWN (NAMED FOR
C                     NUMBER OF DEGREES OF FREEDOM)
C            NUMNOD = NUMBER OF NODES
C             OMEGA = RELAXATION FACTOR
C           OMOMEGA = 1 - OMEGA
C
C  USAGE:
C         FOR EACH TIME STEP, THE RIGHT-HAND-SIDE VECTOR IS
C         COMPUTED USING THE VALUES OF HEAD FOR THE PREVIOUS TIME
C         STEP, AND THE MODIFIED COMBINED CONDUCTION AND CAPACITANCE MATRIX,
C         RELAXATION FACTOR, AND TIME STEP INTERVAL FOR THAT TIME STEP
C
C       SUBROUTINES CALLED:
C         LOC
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
C
        IF (T .GT. TIME(IGT)) IGT = IGT + 1
        T = T + DELTAT(IDT)
        IF (T .GT. TIME(IGTDT)) IGTDT = IGTDT + 1
        I = 0
        DO 10 J = 1, NUMNOD
          IF (ICH(J) .EQ. 0) THEN
            I = I + 1
            B(I) = FC(I) + DELTAT(IDT) * (OMOMEGA * GT(IGT) * FLUX(J)
     1             + OMEGA * GT(IGTDT) * FLUX(J))
          ENDIF
 10     CONTINUE
        J1 = 1
        J2 = SBW
        DO 60 I = 1, NDOF
          J = 0
          DO 20 K = 1, NUMNOD
            IF (ICH(K) .EQ. 0) THEN
              J = J + 1
              IF (J .EQ. J1) GOTO 30
            ENDIF
```

```
20       CONTINUE
 30       K = K - 1

          DO 50 J = J1, J2
 40         K = K + 1
            IF (ICH(K) .NE. 0) GOTO 40
            CALL LOC(I,J,IJ,NDOF,SBW,SYMM)
            B(I) = B(I) + B1(IJ) * X(K)
 50       CONTINUE
          IF (I .GE. SBW) J1 = J1 + 1
          IF (J2 .LT. NDOF) J2 = J2 + 1
 60     CONTINUE
        RETURN
        END
```

```
       SUBROUTINE VELOCITY
C**********************************************************************
C
C         TO COMPUTE THE COMPONENTS OF APPARENT GROUNDWATER
C         VELOCITY FOR EACH ELEMENT IN THE MESH
C
C  INPUT:
C         NONE
C
C  OUTPUT:
C         THE COMPONENTS OF APPARENT GROUNDWATER VELOCITY ARE
C         WRITTEN TO THE USER-DEFINED FILE ASSIGNED TO UNIT
C         "OUTF".
C
C  DEFINITIONS OF VARIABLES:
C                DIM = COORDINATE SYSTEM TYPE
C                  E = ELEMENT NUMBER
C         ELEMTYP(I) = ELEMENT TYPE FOR ELEMENT I
C             NUMELM = NUMBER OF ELEMENTS IN THE MESH
C              V1(I) = APPARENT GROUNDWATER VELOCITY IN X
C                      COORDINATE DIRECTION (DIM=1, 2, OR 3)
C                    = APPARENT GROUNDWATER VELOCITY IN R
C                      COORDINATE DIRECTION (DIM=4)
C              V2(I) = UNUSED (DIM=1)
C                    = APPARENT GROUNDWATER VELOCITY IN Y
C                      COORDINATE DIRECTION (DIM=2 OR 3)
C                    = APPARENT GROUNDWATER VELOCITY IN Z
C                      COORDINATE DIRECTION (DIM=4)
C              V3(I) = UNUSED (DIM=1, 2, OR 4)
C                    = APPARENT GROUNDWATER VELOCITY IN Z
C                      COORDINATE DIRECTION (DIM=3)
C
C  USAGE:
C         THE COMPONENTS OF APPARENT GROUNDWATER VELOCITY ARE
C         COMPUTED IN A SET OF SUBROUTINES, ONE SUBROUTINE
C         FOR EACH ELEMENT TYPE.
C
C       SUBROUTINES CALLED:
C         VBAR2,VTRI3,VREC4,VQUA4
C
C**********************************************************************
        INCLUDE  'COMALL.IN'
```

```
C
        WRITE(OUTF,10)
 10     FORMAT(//70('*')//11X,'COMPUTED VALUES OF APPARENT ',
     1         'GROUNDWATER VELOCITY'/11X,48('-'))
        IF (DIM .EQ. 1) THEN
          WRITE(OUTF,20) '                ','VX',' ',' '
        ELSEIF (DIM .EQ. 2) THEN
          WRITE(OUTF,20) '        ','VX','VY',' '
        ELSEIF (DIM .EQ. 3) THEN

WRITE(OUTF,20) ' ','VX','VY','VZ'
        ELSEIF (DIM .EQ. 4) THEN
          WRITE(OUTF,20) '        ','VR','VZ',' '
        ENDIF
 20     FORMAT(/7X,A,'ELEMENT',10X,A,2(13X,A)/)
C       COMPUTE THE COMPONENTS OF APPARENT GROUNDWATER VELOCITY
C       FOR EACH ELEMENT
        DO 40 E = 1, NUMELM
          IF (ELEMTYP(E) .EQ. 1) THEN
C           ELEMENT IS A LINEAR BAR
            CALL VBAR2(E,V1(E))
          ELSEIF (ELEMTYP(E) .EQ. 4) THEN
C           ELEMENT IS A LINEAR TRIANGLE
            CALL VTRI3(E,V1(E),V2(E))
          ELSEIF (ELEMTYP(E) .EQ. 5) THEN
C           ELEMENT IS A LINEAR RECTANGLE
            CALL VREC4(E,V1(E),V2(E))
          ELSEIF (ELEMTYP(E) .EQ. 6) THEN
C           ELEMENT IS A LINEAR QUADRILATERAL
            CALL VQUA4(E,V1(E),V2(E))
          ENDIF
          IF (DIM .EQ. 1) THEN
            WRITE(OUTF,30) '                ',E,V1(E)
          ELSEIF (DIM .EQ. 2) THEN
            WRITE(OUTF,30) '        ',E,V1(E),V2(E)
          ELSEIF (DIM .EQ. 3) THEN
            WRITE(OUTF,30) ' ',E,V1(E),V2(E),V3(E)
          ELSE
            WRITE(OUTF,30) '        ',E,V1(E),V2(E)
          ENDIF
 30       FORMAT(7X,A,I5,4X,1P3E15.6)
 40     CONTINUE
        RETURN
        END
```

```
  SUBROUTINE VBAR2(E,VXE)
C**********************************************************************
C
C       PURPOSE:
C         TO COMPUTE APPARENT GROUNDWATER VELOCITY FOR A
C         ONE-DIMENSIONAL, LINEAR BAR ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               DHDX = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO X
C                  E = ELEMENT NUMBER
C                KXE = HYDRAULIC CONDUCTIVITY IN X COORDINATE DIRECTION
C                 LE = ELEMENT LENGTH
```

```
C                 VXE = APPARENT GROUNDWATER VELOCITY IN
C                       X COORDINATE DIRECTION
C          X(IN(E,I)) = COMPUTED HEAD FOR NODE I, ELEMENT E
C         X1(IN(E,I)) = X COORDINATE FOR NODE I, ELEMENT E
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
      REAL KXE,LE
C
      KXE = PROP(E,1)
      LE  = X1(IN(E,2)) - X1(IN(E,1))
      DHDX = (X(IN(E,2)) - X(IN(E,1))) / LE
      VXE = -KXE * DHDX
      RETURN
      END



SUBROUTINE VTRI3(E,VXE,VYE)
C*********************************************************************
C
C       PURPOSE:
C         TO COMPUTE COMPONENTS OF APPARENT GROUNDWATER
C         VELOCITY FOR A TWO-DIMENSIONAL, LINEAR TRIANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C                DHDX = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO X
C                DHDY = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO Y
C             DNDX(I) = PARTIAL DERIVATE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO X FOR NODE I
C             DNDY(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                       FUNCTION WITH RESPECT TO Y FOR NODE I
C                   E = ELEMENT NUMBER
C                 KXE = HYDRAULIC CONDUCTIVITY IN X COORDINATE DIRECTION
C                 KYE = HYDRAULIC CONDUCTIVITY IN Y COORDINATE DIRECTION
C                 VXE = APPARENT GROUNDWATER VELOCITY IN
C                       X COORDINATE DIRECTION
C                 VYE = APPARENT GROUNDWATER VELOCITY IN
C                       Y COORDINATE DIRECTION
C          X(IN(E,I)) = COMPUTED HEAD FOR NODE I, ELEMENT E
C         X1(IN(E,I)) = X COORDINATE FOR NODE I, ELEMENT E
C         X2(IN(E,I)) = Y COORDINATE FOR NODE I, ELEMENT E
C
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
      REAL DNDX(3),DNDY(3),KXE,KYE
C
      KXE = PROP(E,1)
      KYE = PROP(E,2)
      AE2 = X1(IN(E,2)) * X2(IN(E,3)) + X1(IN(E,1)) * X2(IN(E,2)) +
     1      X2(IN(E,1)) * X1(IN(E,3)) - X2(IN(E,3)) * X1(IN(E,1)) -
     2      X1(IN(E,3)) * X2(IN(E,2)) - X1(IN(E,2)) * X2(IN(E,1))
      DNDX(1) = (X2(IN(E,2)) - X2(IN(E,3))) / AE2
      DNDX(2) = (X2(IN(E,3)) - X2(IN(E,1))) / AE2
      DNDX(3) = (X2(IN(E,1)) - X2(IN(E,2))) / AE2
      DNDY(1) = (X1(IN(E,3)) - X1(IN(E,2))) / AE2
      DNDY(2) = (X1(IN(E,1)) - X1(IN(E,3))) / AE2
      DNDY(3) = (X1(IN(E,2)) - X1(IN(E,1))) / AE2
      DHDX = 0.
      DHDY = 0.
      DO 20 I = 1, 3
        DHDX = DHDX + DNDX(I) * X(IN(E,I))
```

```
      DHDY = DHDY + DNDY(I) * X(IN(E,I))
   20 CONTINUE
      VXE = -KXE * DHDX
      VYE = -KYE * DHDY
      RETURN
      END
```

```
SUBROUTINE VREC4(E,VXE,VYE)
C*********************************************************************
C
C       PURPOSE:
C         TO COMPUTE COMPONENTS OF APPARENT GROUNDWATER
C         VELOCITY FOR A TWO-DIMENSIONAL, LINEAR RECTANGLE ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C               DHDX = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO X
C               DHDY = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO Y
C            DNDX(I) = PARTIAL DERIVATE OF INTERPOLATION
C                      FUNCTION WITH RESPECT TO X FOR NODE I
C            DNDY(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                      FUNCTION WITH RESPECT TO Y FOR NODE I
C                  E = ELEMENT NUMBER
C                KXE = HYDRAULIC CONDUCTIVITY IN X COORDINATE DIRECTION
C                KYE = HYDRAULIC CONDUCTIVITY IN Y COORDINATE DIRECTION
C                VXE = APPARENT GROUNDWATER VELOCITY IN
C                      X COORDINATE DIRECTION
C                VYE = APPARENT GROUNDWATER VELOCITY IN
C                      Y COORDINATE DIRECTION
C         X(IN(E,I)) = COMPUTED HEAD FOR NODE I, ELEMENT E
C        X1(IN(E,I)) = X COORDINATE FOR NODE I, ELEMENT E
C        X2(IN(E,I)) = Y COORDINATE FOR NODE I, ELEMENT E
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
      REAL DNDX(4),DNDY(4),KXE,KYE
C
      KXE = PROP(E,1)
      KYE = PROP(E,2)
      AE = ABS(X2(IN(E,1)) - X2(IN(E,3))) / 2.
      BE = ABS(X1(IN(E,1)) - X1(IN(E,3))) / 2.

      DNDX(1) = - 1. / (2.*BE)
      DNDX(2) = -DNDX(1)
      DNDX(3) =  0
      DNDX(4) =  0
      DNDY(1) = - 1. / (2.*AE)
      DNDY(2) =  0
      DNDY(3) =  0
      DNDY(4) = -DNDY(1)

      DHDX = 0.
      DHDY = 0.
      DO 10 I = 1, 4
        DHDX = DHDX + DNDX(I) * X(IN(E,I))
        DHDY = DHDY + DNDY(I) * X(IN(E,I))
   10 CONTINUE
```

```
      VXE = -KXE * DHDX
      VYE = -KYE * DHDY
      RETURN
      END
```

```
      SUBROUTINE VQUA4(E,VXE,VYE)
C*********************************************************************
C
C         TO COMPUTE COMPONENTS OF APPARENT GROUNDWATER
C         VELOCITY FOR A TWO-DIMENSIONAL, LINEAR QUADRILATERAL
C         ELEMENT
C
C       DEFINITIONS OF VARIABLES:
C             DETJAC = DETERMINANT OF JACOBIAN MATRIX
C               DHDX = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO X
C               DHDY = PARTIAL DERIVATIVE OF HEAD WITH RESPECT TO Y
C           DNDXI(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                      FUNCTION WITH RESPECT TO XI FOR NODE I
C            DNDX(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                      FUNCTION WITH RESPECT TO X FOR NODE I
C          DNDETA(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                      FUNCTION WITH ESPECT TO ETA FOR NODE I
C            DNDY(I) = PARTIAL DERIVATIVE OF INTERPOLATION
C                      FUNCTION WITH RESPECT TO Y FOR NODE I
C                  E = ELEMENT NUMBER
C           JAC(I,J) = JACOBIAN MATRIX
C        JACINV(I,J) = INVERSE OF JACOBIAN MATRIX
C                KXE = HYDRAULIC CONDUCTIVITY IN X COORDINATE DIRECTION
C                KYE = HYDRAULIC CONDUCTIVITY IN Y COORDINATE DIRECTION
C                VXE = APPARENT GROUNDWATER VELOCITY IN
C                      X COORDINATE DIRECTION
C                VYE = APPARENT GROUNDWATER VELOCITY IN
C                      Y COORDINATE DIRECTION
C         X(IN(E,I)) = COMPUTED HEAD FOR NODE I, ELEMENT E
C        X1(IN(E,I)) = X COORDINATE FOR NODE I, ELEMENT E
C        X2(IN(E,I)) = Y COORDINATE FOR NODE I, ELEMENT E
C
C*********************************************************************
        INCLUDE  'COMALL.IN'
        REAL JAC(2,2),JACINV(2,2),DNDXI(4),DNDX(4),
     1       DNDETA(4),DNDY(4),SIGN1(4),SIGN2(4),KXE,KYE
        DATA SIGN1/-1.,1.,1.,-1./
        DATA SIGN2/-1.,-1.,1.,1./
C
        KXE = PROP(E,1)
        KYE = PROP(E,2)

        DO 20 I = 1, 2
          DO 10 J = 1, 2
            JAC(I,J) = 0.
 10       CONTINUE
 20     CONTINUE

        DO 30 I = 1, 4
          DNDXI(I)  = 0.25 * SIGN1(I)
          DNDETA(I) = 0.25 * SIGN2(I)
```

```
 30     CONTINUE

        DO 40 I = 1, 4
          JAC(1,1) = JAC(1,1) + DNDXI(I) * X1(IN(E,I))
          JAC(1,2) = JAC(1,2) + DNDXI(I) * X2(IN(E,I))
          JAC(2,1) = JAC(2,1) + DNDETA(I) * X1(IN(E,I))
          JAC(2,2) = JAC(2,2) + DNDETA(I) * X2(IN(E,I))
 40     CONTINUE

        DETJAC = JAC(1,1) * JAC(2,2) - JAC(1,2) * JAC(2,1)
        JACINV(1,1) =  JAC(2,2) / DETJAC
        JACINV(1,2) = -JAC(1,2) / DETJAC

JACINV(2,1) = -JAC(2,1) / DETJAC
        JACINV(2,2) =  JAC(1,1) / DETJAC

        DO 50 I = 1, 4
          DNDX(I) = JACINV(1,1) * DNDXI(I) + JACINV(1,2) * DNDETA(I)
          DNDY(I) = JACINV(2,1) * DNDXI(I) + JACINV(2,2) * DNDETA(I)
 50     CONTINUE

        DHDX = 0.
        DHDY = 0.

        DO 60 I = 1, 4
          DHDX = DHDX + DNDX(I) * X(IN(E,I))
          DHDY = DHDY + DNDY(I) * X(IN(E,I))
 60     CONTINUE

        VXE = -KXE * DHDX
        VYE = -KYE * DHDY
        RETURN
        END




        SUBROUTINE SOLVE(NDOF,SBW,SYMM,M,B,X)
C*********************************************************************
C
C          SUBROUTINES DECOMP AND SOLVE SOLVE A SYSTEM OF
C          LINEAR EQUATIONS OF THE FORM
C                      [M] {X} = {B}
C          WHERE [M] IS A BANDED MATRIX OF KNOWN COEFFICIENTS
C          (SYMMETRIC OR NONSYMMETRIC), {X} ARE THE UNKNOWNS,
C          AND {B} IS A VECTOR OF KNOWN VALUES
C
C  INPUT:
C          NONE
C
C  OUTPUT:
C          NONE
C
C  DEFINITIONS OF VARIABLES:
C           B(I) = VECTOR OF KNOWN VALUES
C          M(IJ) = MATRIX OF KNOWN COEFFICIENTS IN VECTOR STORAGE
C           NDOF = NUMBER OF UNKNOWN VALUES {X}
C            SBW = SEMI-BANDWIDTH OF [M]
C           SYMM = LOGICAL VARIABLE
C                = 'TRUE' IF [M] IS SYMMETRIC
```

```
C                = 'FALSE' IF [M] IS NONSYMMETRIC
C           X(I) = VECTOR OF UNKNOWN VALUES TO BE COMPUTED
C
C  USAGE:
C          SUBROUTINE DECOMP PERFORMS TRIANGULAR DECOMPOSITION
C          ON THE MATRIX OF KNOWN COEFFICIENTS IN VECTOR MATRIX
C          STORAGE, {M}.  THE RESULTING UPPER-, AND LOWER-
C          TRIANGULAR MATRICES ARE STORED IN {M} (THE ORIGINAL
C          CONTENTS OF {M} ARE OVERWRITTEN DURING THE
C          DECOMPOSITION PROCESS).  SUBROUTINE SOLVE SOLVES FOR
C          VALUES OF THE UNKNOWNS BY BACKWARD SUBSTITUTION.  ONCE
C          {M} HAS BEEN DECOMPOSED SOLVE CAN BE USED TO OBTAIN
C          VALUES OF {X} FOR ANY NUMBER OF DIFFERENT VECTORS {B}.
C
C       SUBROUTINES CALLED:
C         LOC
C
C**********************************************************************
        INTEGER NDOF,SBW
        LOGICAL SYMM
        REAL M(1),B(1),X(1)
C
        DO 10 I = 1, NDOF
          X(I) = B(I)
 10     CONTINUE
        IF (SYMM) THEN
C         M IS A SYMMETRIC MATRIX
          K2 = SBW
          IK = 0
          DO 30 I = 1, NDOF
            DO 20 K = I, K2
              IK = IK + 1
              IF (K .EQ. I) THEN
                X(K) = X(K) / M(IK)
              ELSE
                X(K) = X(K) - M(IK) * X(I)
              ENDIF
 20         CONTINUE
            IF (K2 .LT. NDOF) K2 = K2 + 1
 30       CONTINUE
          K2 = 0
          DO 50 I = NDOF, 1, -1
            IF (K2 .GT. 0) THEN
              DO 40 K = (I + K2), (I + 1), -1
                X(I) = X(I) - M(IK) * X(K)
                IK = IK - 1
 40           CONTINUE
            ENDIF
            X(I) = X(I) / M(IK)
            IK = IK - 1
            IF (K2 .LT. (SBW - 1)) K2 = K2 + 1
 50       CONTINUE
        ELSE
C         M IS A NONSYMMETRIC MATRIX
          X(1) = X(1) / M(1)
          IF (NDOF .GT. 1) THEN
            K2 = 1
            DO 70 I = 2, NDOF
              IF (I .GT. SBW) K2 = K2 + 1
              CALL LOC(I,I,II,NDOF,SBW,SYMM)
              IF (I .GT. K2) THEN
```

```
                IK = II - 1
                DO 60 K = (I - 1), K2, -1
                  X(I) = X(I) - M(IK) * X(K)
                  IK = IK - 1
60              CONTINUE
              ENDIF
              X(I) = X(I) / M(II)
70          CONTINUE
          ENDIF
          J = NDOF - SBW + 1
          K2 = NDOF
          IF (NDOF .GT. 1) THEN
            DO 90 I = (NDOF - 1), 1, -1
              IF (I .LT. J) K2 = K2 - 1
              IF (I .LT. K2) THEN
                CALL LOC(I,I,II,NDOF,SBW,SYMM)
                IK = II + 1
                DO 80 K = (I + 1), K2
                  X(I) = X(I) - M(IK) * X(K)
                  IK = IK + 1

80              CONTINUE
              ENDIF
90          CONTINUE
          ENDIF
        ENDIF
        RETURN
        END
```

A.3 Data and Output Files of the Example

Several examples were performed using the COMPBAR computer program. One of the examples of the data files and output files are presented next.

A.3.1 Data File of the Example 1

```
GOOD CONTACT CONDITION
2              DIM
5              # OF HOLES
1,-2           UNITS  (0-m, 1-cm, 2-ft, & 3-in),   INITIAL PRESSURE FOR TOP NODES
400            X COORDINATE LOCATION FOR  THE FIRST HOLE !
0,0            TYPE & CONTACT OF THE HOLE !  REPEAT FOR EVERY HOLE
0.00564,0.3,1.3E-9     R0,  hw,  & Kw  ALL IN  m & m/sec!
900            X COORDINATE LOCATION FOR  THE SECOND HOLE !
0,0            TYPE & CONTACT OF THE HOLE !
0.00564,0.3,1.3E-9     R0,  hw,  & Kw  ALL IN  m & m/sec!
1400           X COORDINATE LOCATION FOR  THE THIRD HOLE !
0,0            TYPE & CONTACT OF THE HOLE !
0.00564,0.3,1.3E-9     R0,  hw,  & Kw  ALL IN  m & m/sec!
2000           X COORDINATE LOCATION FOR  THE FOURTH HOLE !
0,0            TYPE & CONTACT OF THE HOLE !
0.00564,0.3,1.3E-9     R0,  hw,  & Kw  ALL IN  m & m/sec!
2600           X COORDINATE LOCATION FOR  THE FIFTH HOLE !
0,0            TYPE & CONTACT OF THE HOLE !
0.00564,0.3,1.3E-9     R0,  hw,  & Kw  ALL IN  m & m/sec!
1     1     0     0          NODE 1,     INC,    XYZ COORDINATE
12    1     0     110          "           "          "
17    1     0     120          "           "          "
18    1     200   0            "           "          "
29    1     200   110          "           "          "
34    1     200   120          "           "          "
35    1     400   0            "           "          "
46    1     400   110          "           "          "
51    1     400    120         "           "          "
52    1     600    0           "           "          "
63    1     600   110          "           "          "
68    1     600   120          "           "          "
69    1     800   0            "           "          "
80    1     800   110          "           "          "
85    1     800   120          "           "          "
86    1     820   0            "           "          "
97    1     820   110          "           "          "
102   1     820   120          "           "          "
103   1     840   0            "           "          "
114   1     840   110          "           "          "
```

119	1	840	120	"	"	"
120	1	860	0	"	"	"
131	1	860	110	"	"	"
136	1	860	120	"	"	"
137	1	880	0	"	"	"
148	1	880	110	"	"	"
153	1	880	120	"	"	"
154	1	900	0	"	"	"
165	1	900	110	"	"	"
170	1	900	120	"	"	"
171	1	920	0	"	"	"
182	1	920	110	"	"	"
187	1	920	120	"	"	"
188	1	940	0	"	"	"
199	1	940	110	"	"	"
204	1	940	120	"	"	"
205	1	960	0	"	"	"
216	1	960	110	"	"	"
221	1	960	120	"	"	"
222	1	980	0	"	"	"
233	1	980	110	"	"	"
238	1	980	120	"	"	"
239	1	1000	0	"	"	"
250	1	1000	110	"	"	"
255	1	1000	120	"	"	"
256	1	1200	0	"	"	"
267	1	1200	110	"	"	"
272	1	1200	120	"	"	"
273	1	1400	0	"	"	"
284	1	1400	110	"	"	"
289	1	1400	120	"	"	"
290	1	1600	0	"	"	"
301	1	1600	110	"	"	"
306	1	1600	120	"	"	"
307	1	1800	0	"	"	"
318	1	1800	110	"	"	"
323	1	1800	120	"	"	"
324	1	2000	0	"	"	"
335	1	2000	110	"	"	"
340	1	2000	120	"	"	"
341	1	2200	0	"	"	"
352	1	2200	110	"	"	"
357	1	2200	120	"	"	"
358	1	2400	0	"	"	"
369	1	2400	110	"	"	"
374	1	2400	120	"	"	"

```
375    1      2600   0               "            "            "
386    1      2600   110             "            "            "
391    1      2600   120             "            "            "
392    1      2800   0               "            "            "
403    1      2800   110             "            "            "
408    1      2800   120             "            "            "
409    1      3000   0               "            "            "
420    1      3000   110             "            "            "
425    1      3000   120             "            "            "
-1    -1   -1  -1         TO END READING NODES DATA
1       5 17   1  18  19   2   EL DATA: ELM, TYPE, NODE INC., #OF ELEM
24      5 17                     392  409  410  393
25      5 17   2  19  20   3
48      5 17                     393  410  411  394
49      5 17   3  20  21   4
72      5 17                     394  411  412  395
73      5 17   4  21  22   5
96      5 17                     395  412  413  396
97      5 17   5  22  23   6
120     5 17                     396  413  414  397
121     5 17   6  23  24   7
144     5 17                     397  414  415  398
145     5 17   7  24  25   8
168     5 17                     398  415  416  399
169     5 17   8  25  26   9
192     5 17                     399  416  417  400
193     5 17   9  26  27  10
216     5 17                     400  417  418  401
217     5 17  10  27  28  11
240     5 17                     401  418  419  402
241     5 17  11  28  29  12
264     5 17                     402  419  420  403
265     5 17  12  29  30  13
288     5 17                     403  420  421  404
289     5 17  13  30  31  14
312     5 17                     404  421  422  405
313     5 17  14  31  32  15
336     5 17                     405  422  423  406
337     5 17  15  32  33  16
360     5 17                     406  423  424  407
361     5 17  16  33  34  17
384     5 17                     407  424  425  408
-1     -1  -1   -1    -1    -1   -1        TO END READING ELEMENT DATA
1         1          ELEMENT #,  MATERIAL SET #
72        1                 "      "
73       2               "                "
```

```
384       2                  "                      "
-1      -1            TO END READING
9                       # PROPERTIES TO BE READ = DIM+7
1 3      820.8     820.8     0.0  0.0307 3.9 0.7436 0.435 0.069 0.435
2 3      0.01123   0.01123   0.0  0.0249 1.6 0.3750 0.495 0.175 0.495
1           0.15       #1 ALWAYS,    ASSUMED INITIAL VOLUMETRIC MOISTURE
CONTENT
2          0.37    "                      "
17,425,17       FIRST NODE # ON TOP OF GEO, LAST #, INC
-1      -1    NO SPECIFIED VALUES OF PRESSURE HEAD
-1      -1    NO SPECIFIED VALUES OF GW FLOW
1.
1       5
9       10
20      30
34      60
40      90
-1     -1
0       1
5          1
1400    1
-1     -1
1         -75      1ST ELEM, INITIAL CONDITION
425     -75      LAST ELEM, INTIAL CONDITION
-1     -1
6                   TO WRITE OUT THE PRESS. & COORD OF NODES
TEST2.PRN
-1
TEST2
```

A.3.2 Output of the Example 1

```
GOOD CONTACT CONDITION
 ************************************
 ** QRATE =   3.7572079E-08 m^3/sec **
 HOLE NUMBER      RADIUS OF WETTED AREA
         1          136.4079
         2          136.4079
         3          136.4079
         4          136.4079
         5          136.4079
 ************************************
  NODE                 NODAL COORDINATES
 NUMBER               X                 Y
 ------     -------------------- --------------------
   1            0.0000            0.0000
   2            0.0000           10.0000
   3            0.0000           20.0000
   .              .
   .                .
   .                .
  422         3000.0000          114.0000
  423         3000.0000          116.0000
  424         3000.0000          118.0000
  425         3000.0000          120.0000


 ELEMENT    ELEMENT
   NO.        TYPE          NODE NUMBERS
 -------    -------        ------------
    1          5         1   18   19   2
    2          5        18   35   36   19
    3          5        35   52   53   36
    4          5        52   69   70   53
    5          5        69   86   87   70
    .          .
    .          .
    .          .
   381          5        356  373  374  357
   382          5        373  390  391  374
   383          5        390  407  408  391
   384          5        407  424  425  408

 ELEMENT
   NO.      MATERIAL SET NUMBER
 -------    --------------------
```

```
  1              1
  2              1
  3              1
  4              1
  5              1
  .              .
  .              .
  .              .
381              2
382              2
383              2
384              2

MATERIAL
 SET NO.                MATERIAL  PROPERTIES
--------             --------------------

# OF NODES WITH SPECIFIED  PRESSURE HEAD        =  25


# OF NODES WITH SPECIFIED  GROUNDWATER FLOW      =   0

OMEGA =        1.0000


START      END       DELTA T
-----      -----     -----------
  1         1          5.0000
  2         9         10.0000
 10        20          30.0000
 21        34          60.0000
 35        40          90.0000
      TOTAL TIME =     1795.0000

     TIME T         G(T)
    --------       ------
      0.0000       1.0000
      5.0000       1.0000
   1795.0000        1.0000


INITIAL VALUES OF  PRESSURE HEAD
--------------------------------------

NODE NO.         PRESSURE HEAD
--------       --------------------
```

1	-75.0000
2	-75.0000
3	-75.0000
.	.
.	.
.	.
423	-75.0000
424	-75.0000
425	-2.0000*

* = SPECIFIED

****

COMPUTED VALUES OF PRESSURE HEAD

NODE NO.	PRESSURE HEAD
1	-45.5033
2	-45.5033
3	-45.5033
.	.
.	
423	-2.8894
424	-2.4394
425	-2.0000*

* = SPECIFIED

*** RESULTS FOR TIME = 3740.00 ***

****

COMPUTED VALUES OF APPARENT GROUNDWATER VELOCITY

ELEMENT	VX	VY
1	-5.264343E-03	-1.175029E+01
2	-8.205965E-03	-1.199063E+01
3	-2.613687E-03	-1.218293E+01
.	.	
.	.	
.	.	
381	-2.754702E-07	-8.445765E-03

382	-1.754449E-03	-1.221595E-02
383	1.751590E-03	-1.645107E-02
384	1.210343E-05	-8.171616E-03

*** RESULTS FOR TIME = 3740.00 ***

ELEM #	K	C	FLOW RATE	MOISTURE
1	1.1750E+01	5.7221E-03	-1.0767E+04	1.8330E-01
2	1.1990E+01	5.7604E-03	-1.0958E+04	1.8401E-01
3	1.2182E+01	5.7905E-03	-1.1110E+04	1.8457E-01
.	.			
.	.			
.	.			
381	7.7388E-03	8.0176E-04	-1.6173E+00	4.9395E-01
382	1.1230E-02	0.0000E+00	1.5412E+01	4.9500E-01
383	1.1230E-02	0.0000E+00	1.5426E+01	4.9500E-01
384	7.6921E-03	8.1324E-04	-1.6474E+00	4.9390E-01

VITA

Mustafa Eftelioglu was born in Manisa, Turkey on January, 02, 1954. He attended Geology, Sciences Faculty, Aegean University in Izmir, Turkey and received the Bachelor of Science in Geology in 1978. After taking extra engineering courses, he received a diploma in Geological Engineering from the same university in 1979. He completed a Masters degree Program in 1983 in Sedimentology at the Institute of Marine Science and Technology, Dokuz Eylul University, Izmir, Turkey. He has participated about 50 scientific projects at the Institute of Marine Sciences and Technolgoy related with Oceanograpy and Geotechniques. He has also been teaching American Nato soldiers at Maryland University in Izmir for six years.